conditions will depart simultaneously from the infinite reservoir curve, regardless of the nature of the outer boundary. The results in Figs. 2 and 3 indicate that for practical purposes this observation also holds for all cases except $x_e/x_f = 1$. This implies that a limiting statement can be made concerning the drainage volume for a fractured well that does not indicate a drainage boundary effect for both closed and constant-pressure boundary cases provided $x_e/x_f \neq 1$; that is, if the fracture does not extend to the outer boundary.

Comparison of Figs. 2 and 3 for the closed and constant-pressure cases for $x_e/x_f = 1$ indicates one important difference. The pressure drops are identical for the uniform-flux and infinite-conductivity cases for the closed reservoir, whereas for the constant-pressure case this is not so. These results are caused by the influence of the outer boundary. If $x_e/x_f = 1$, no pressure gradients parallel to the fracture plane exist in a closed reservoir; in the constant-pressure case this is not so.

## AVERAGE RESERVOIR PRESSURE

In the case of a closed reservoir, the average reservoir pressure in an area drained by a well may be obtained from a simple materials balance. For a well in a closed system producing at a constant rate, the dimensionless average pressure obtained by a materials balance is given by

$$\bar{p}_D\,(t_{DA}) = \frac{kh}{141.2\,qB\mu}[p_i - \bar{p}\,(t)] = 2\,\pi\,t_{DA}\ . \qquad (8)$$

For the system under study here, the average reservoir pressure cannot be obtained readily from a materials balance unless the quantity of fluid crossing the constant pressure boundary is known. Dimensionless average reservoir pressures may be calculated, however, by integrating Eq. 1 over the drainage area. These are shown in Table 3 and are accurate to five digits.

The $\bar{p}_D\,(t_{DA})$ values presented in Table 3 represent important new information. Theoretically, if a well producing from a *bounded* system is shut in, then the wellbore pressure will stabilize at the mean pressure of the area drained at the instant the well is shut in. However, in the system examined here the final pressure will not stabilize at the mean pressure of the system but at the pressure corresponding to that of the outer boundary, namely $p_i$. Furthermore, the average reservoir pressure during buildup changes as fluid crosses the reservoir boundary. In most engineering applications such as material-balance computations, the average reservoir pressure at the *instant* of shut-in and not at the *end* of the buildup period is required. Once $p_i$ is determined, the information shown in Table 3 can be used to calculate the average reservoir pressure. This will be examined further in considering aspects of buildup.

At this point, one qualifying remark about Table 3 is necessary. Strictly speaking, the results presented by Table 3 are rigorously correct only for the uniform-flux case. As pointed out by Gringarten *et al.*[1], the flux distribution per unit area of the fracture for the infinite-conductivity fracture changes with time until the flux distribution reaches a stabilized condition at long times (Fig. 2 of Ref. 1). This distribution is different from the uniform-flux case and thus the areal pressure distribution around the fracture would be different. To our knowledge, no *simple method* exists for determining pressure distribution surrounding an infinite-conductivity vertical fracture, although the pressure at the fracture may be readily determined using the procedure of Gringarten *et al*. Thus, the results in Table 3 should be understood to be somewhat approximate, especially when applying

$t_D(x_f) = (T_{DA}) \times (4)(x_e/x_f)^2$

TABLE 2 — DIMENSIONLESS WELLBORE PRESSURE DROP FOR A UNIFORM-FLUX VERTICAL FRACTURE AT THE CENTER OF A CONSTANT-PRESSURE SQUARE

| Dimensionless Time, $t_{DA}$ — Fracture Penetration Ratio, $x_e/x_f$ | Dimensionless Wellbore Pressure Drop, $p_{wD}$* | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 1.5 | 2 | 3 | 5 | 7 | 10 | 15 |
| 0.01000 | 0.35447 | 0.53060 | | | | | | |
| 0.02000 | 0.50001 | 0.73915 | | | | | | |
| 0.03000 | 0.60695 | 0.88494 | 1.11788 | | | | | |
| 0.04000 | 0.69069 | 0.99613 | 1.24159 | 1.61427 | 2.10733 | | | |
| 0.05000 | 0.75790 | 1.08433 | 1.33856 | 1.71813 | 2.21496 | 2.54761 | 2.90216 | |
| 0.06000 | 0.81250 | 1.15558 | 1.41647 | 1.80114 | 2.30071 | 2.63412 | 2.98909 | 3.40543 |
| 0.07000 | 0.85711 | 1.21364 | 1.47981 | 1.86846 | 2.37013 | 2.70414 | 3.05942 | 3.47593 |
| 0.08000 | 0.89364 | 1.26114 | 1.53157 | 1.92341 | 2.42677 | 2.76125 | 3.11678 | 3.53343 |
| 0.09000 | 0.92360 | 1.30007 | 1.57398 | 1.96841 | 2.47313 | 2.80799 | 3.16372 | 3.58048 |
| 0.10000 | 0.94818 | 1.33201 | 1.60875 | 2.00530 | 2.51113 | 2.84630 | 3.20220 | 3.61905 |
| 0.20000 | 1.04514 | 1.45798 | 1.74589 | 2.15076 | 2.66097 | 2.99736 | 3.35391 | 3.77122 |
| 0.30000 | 1.05861 | 1.47547 | 1.76494 | 2.17096 | 2.68178 | 3.01834 | 3.37498 | 3.79236 |
| 0.40000 | 1.06048 | 1.47790 | 1.76758 | 2.17377 | 2.68467 | 3.02125 | 3.37790 | 3.79529 |
| 0.50000 | 1.06074 | 1.47824 | 1.76795 | 2.17416 | 2.68507 | 3.02166 | 3.37831 | 3.79576 |
| 0.60000 | 1.06078 | 1.47829 | 1.76800 | 2.17421 | 2.68513 | 3.02171 | 3.37837 | 3.79576 |
| > 0.70000 | 1.06079 | 1.47829 | 1.76801 | 2.17422 | 2.68514 | 3.02172 | 3.37837 | 3.79576 |

*Values of $p_{wD}$ for times smaller than that shown here are identical to the closed outer boundary case.

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$TD(xf) = 4(xe/xf)^2 (tDA)$

| Dimensionless Time, tDA - | Dimensionless Wellbore Pressure Drop, pwD | TD(xf) for Fracture Penetration Ratio of 1 |
|---|---|---|
| 0.01000 | 0.35447 | 0.04000 |
| 0.02000 | 0.50001 | 0.08000 |
| 0.03000 | 0.60695 | 0.12000 |
| 0.04000 | 0.69069 | 0.16000 |
| 0.05000 | 0.75790 | 0.20000 |
| 0.06000 | 0.81250 | 0.24000 |
| 0.07000 | 0.85711 | 0.28000 |
| 0.08000 | 0.89364 | 0.32000 |
| 0.09000 | 0.92360 | 0.36000 |
| 0.10000 | 0.94818 | 0.40000 |
| 0.20000 | 1.04514 | 0.80000 |
| 0.30000 | 1.05861 | 1.20000 |
| 0.40000 | 1.06048 | 1.60000 |
| 0.50000 | 1.06074 | 2.00000 |
| 0.60000 | 1.06078 | 2.40000 |
| 0.70000 | 1.06079 | 2.80000 |

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$TD(xf) = 4(xe/xf)^2 (tDA)$

| Dimensionless Time, tDA - | Dimensionless Wellbore Pressure Drop, pwD | TD(xf) for Fracture Penetration Ratio of 1.5 |
|---|---|---|
| 0.01000 | 0.53060 | 0.09000 |
| 0.02000 | 0.73915 | 0.18000 |
| 0.03000 | 0.88494 | 0.27000 |
| 0.04000 | 0.99613 | 0.36000 |
| 0.05000 | 1.08433 | 0.45000 |
| 0.06000 | 1.15558 | 0.54000 |
| 0.07000 | 1.21364 | 0.63000 |
| 0.08000 | 1.26114 | 0.72000 |
| 0.09000 | 1.30007 | 0.81000 |
| 0.10000 | 1.33201 | 0.90000 |
| 0.20000 | 1.45798 | 1.80000 |
| 0.30000 | 1.47547 | 2.70000 |
| 0.40000 | 1.47790 | 3.60000 |
| 0.50000 | 1.47824 | 4.50000 |
| 0.60000 | 1.47829 | 5.40000 |
| 0.70000 | 1.47829 | 3.30000 |

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$TD(xf) = 4(xe/xf)^2 (tDA)$

| Dimensionless Time, tDA - | Dimensionless Wellbore Pressure Drop, pwD | TD(xf) for Fracture Penetration Ratio of 2 |
|---|---|---|
| 0.01000 | | |
| 0.02000 | | |
| 0.03000 | 1.11788 | 0.48000 |
| 0.04000 | 1.24159 | 0.64000 |
| 0.05000 | 1.33856 | 0.80000 |
| 0.06000 | 1.41647 | 0.96000 |
| 0.07000 | 1.47981 | 1.12000 |
| 0.08000 | 1.53157 | 1.28000 |
| 0.09000 | 1.57398 | 1.44000 |
| 0.10000 | 1.60875 | 1.60000 |
| 0.20000 | 1.74589 | 3.20000 |
| 0.30000 | 1.76494 | 4.80000 |
| 0.40000 | 1.76758 | 6.40000 |
| 0.50000 | 1.76795 | 8.00000 |
| 0.60000 | 1.76800 | 9.60000 |
| 0.70000 | 1.76801 | 11.20000 |

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| Dimensionless Time, tDA - | Dimensionless Wellbore Pressure Drop, pwD | $TD(xf) = 4(xe/xf)^2 (tDA)$<br>TD(xf) for Fracture Penetration Ratio of 3 |
|---|---|---|
| 0.01000 | | |
| 0.02000 | | |
| 0.03000 | | |
| 0.04000 | 1.61427 | 1.44000 |
| 0.05000 | 1.71813 | 1.80000 |
| 0.06000 | 1.80114 | 2.16000 |
| 0.07000 | 1.86846 | 2.52000 |
| 0.08000 | 1.92341 | 2.88000 |
| 0.09000 | 1.96841 | 3.24000 |
| 0.10000 | 2.00530 | 3.60000 |
| 0.20000 | 2.15076 | 7.20000 |
| 0.30000 | 2.17096 | 10.80000 |
| 0.40000 | 2.17377 | 14.40000 |
| 0.50000 | 2.17416 | 18.00000 |
| 0.60000 | 2.17421 | 21.60000 |
| 0.70000 | 2.17422 | 25.20000 |

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$TD(xf) = 4(xe/xf)^2 \; (tDA)$

| Dimensionless Time, tDA - | Dimensionless Wellbore Pressure Drop, pwD | TD(xf) for Fracture Penetration Ratio of 5 |
|---|---|---|
| 0.01000 | | |
| 0.02000 | | |
| 0.03000 | | |
| 0.04000 | 2.10733 | 4.00000 |
| 0.05000 | 2.21496 | 5.00000 |
| 0.06000 | 2.30071 | 6.00000 |
| 0.07000 | 2.37013 | 7.00000 |
| 0.08000 | 2.42677 | 8.00000 |
| 0.09000 | 2.47313 | 9.00000 |
| 0.10000 | 2.51113 | 10.00000 |
| 0.20000 | 2.66097 | 20.00000 |
| 0.30000 | 2.68178 | 30.00000 |
| 0.40000 | 2.68467 | 40.00000 |
| 0.50000 | 2.68507 | 50.00000 |
| 0.60000 | 2.68513 | 60.00000 |
| 0.70000 | 2.68514 | 70.00000 |

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$TD(xf) = 4(xe/xf)^2 (tDA)$

| Dimensionless Time, tDA - | Dimensionless Wellbore Pressure Drop, pwD | TD(xf) for Fracture Pentration Ratio of 10 |
|---|---|---|
| 0.01000 | | |
| 0.02000 | | |
| 0.03000 | | |
| 0.04000 | | |
| 0.05000 | 2.90216 | 20.00000 |
| 0.06000 | 2.98909 | 24.00000 |
| 0.07000 | 3.05942 | 28.00000 |
| 0.08000 | 3.11678 | 32.00000 |
| 0.09000 | 3.16372 | 36.00000 |
| 0.10000 | 3.20220 | 40.00000 |
| 0.20000 | 3.35391 | 80.00000 |
| 0.30000 | 3.37498 | 120.00000 |
| 0.40000 | 3.37790 | 160.00000 |
| 0.50000 | 3.37831 | 200.00000 |
| 0.60000 | 3.37837 | 240.00000 |
| 0.70000 | 3.37837 | 280.00000 |