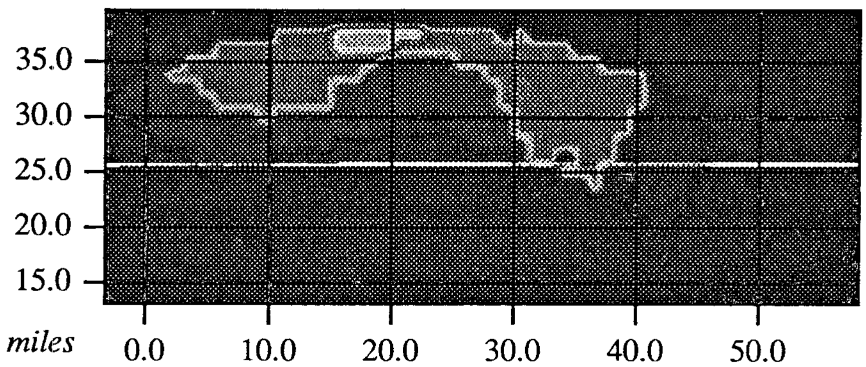
Addendum to Capitan Ground-Water Studies

Michael G. Wallace
Re/Spec Inc., Spring, 1993



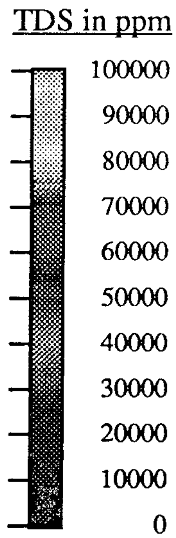


Figure 1. Simulation Including Injection
(1,058 years, no regional gradient)

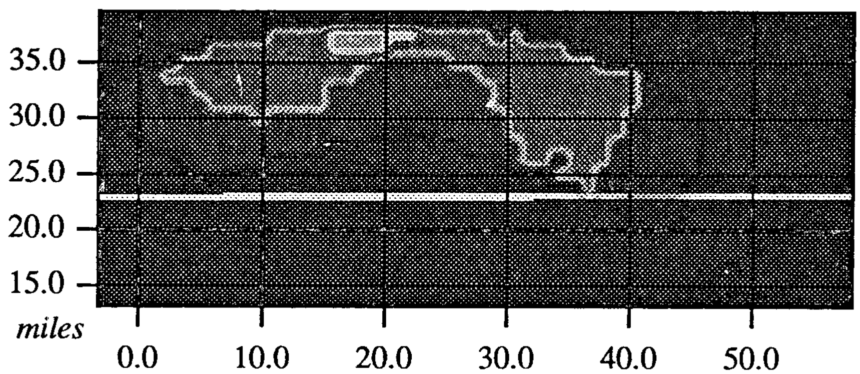


Figure 2. Simulation Without Injection
(1,058 years, no regional gradient)

EXHIBIT 9
PRONGHORN SWD CASE NO.10693
Before Examiner Stogner
May 6, 1993

Addendum to Capitan GroundWater Studies

by Michael G. Wallace
RE/SPEC Inc.

This addendum builds upon the development of the numerical model detailed in the report "Capitan Ground Water Studies" (RE/SPEC, 1993). The numerical model was used to address two scenarios. The first involved an approximation of the regional flow regime (through a steady state run) followed by prediction of the impact from injection upon water quality. The second scenario involved a more conservative assumption of no regional gradient, followed by prediction of the impact from injection upon water quality.

The numerical simulator SUTRA (Saturated-Unsaturated TRAnsport) was utilized in this exercise and is described in the aforementioned report. The model grid consisted of 1,036 nodes, which define 966 four-noded elements. The boundary condition assignments and other input variables, as well as results and conclusions, have also been discussed in the previous report.

During the presentation of the model results to the Office of the New Mexico State Engineer, a number of issues came up. It was agreed that we would respond to several of those comments, including:

1. *That submarine canyons of the Capitan Reef be referred to as "constrictions" to regional flow, rather than as "barriers".*

2. *That another model run be conducted in which no injection occurs whatsoever for approximately 1,000 years (no regional gradient either).* This was brought up due to an earlier discussion in the model report concerning the effects of molecular diffusion upon the model results, with or without injection.

3. *That the implementation of the storage term in SUTRA calculations be explained.*

4. *That the implementation of the equivalent fresh-water head concept into SUTRA be explained.*

The remainder of this addendum consists of responses to the above issues.

As we are in a process of converting over to a new graphic display system, the model results are displayed somewhat differently than in the previous report. The new system involves the display of gray scale and/or color bands for the depiction of the distribution of a paramter (such as TDS) in two dimensions.

Addendum

1. *That submarine canyons of the Capitan Reef be referred to as "constrictions" to regional flow, rather than as "barriers".*

We concur on this point. "Constrictions to flow" is a more appropriate description of what is likely occuring than "barriers to flow".

2. *That another model run be conducted in which no injection occurs for approximately 1,000 years (no regional gradient either).*

In the Assumptions section of the previous report, it was stated:

> "For the second scenario, ___ there is no regional gradient to carry the brines away from the Pecos. From that standpoint, the second scenario is more conservative than the first scenario, since the high TDS water is allowed to diffuse naturally, over time towards the low TDS zones. An ultimate consequence of this initial condition for the second scenario is that if the model were run long enough, even without any brine injection from the proposed position, the high TDS zones would completely invade the low TDS zones, eventually making the average brine concentration greater than 10,000 ppm throughout the entire model domain. Once again the reader is cautioned to take note of this artifact when reviewing the model results."

During the model presentation, it was suggested by the reviewers that an additional run be conducted in order to ensure that all of the ambient brine that appeared in the simulation to be moving towards the Pecos (an almost undetectable quantity) was moving solely due to this diffusion process, and not due to the injection activity.

This additional run has been conducted. The run setup is identical to the previous case (labeled Scenario #2 in the report) in every respect except that no brine injection occurs in the first 50 years of the simulation.

In Figures 1. and 2., regions of solid shades are regions of a constant TDS interval. By visual comparison with, for example, Figure D13 of the previous report, the correspondance in values can be readily determined. Basically, the major bands of shading correspond with the 0, 10000, 20000, 50000, and 100000 ppm zones of previous figures.

Figure 1 shows the TDS distribution for the original Scenario #2 at 1,058 years. The artifact of the injection activity over 1,000 years earlier is still visible as a small light gray patch.

Figure 2 shows the TDS distribution for the new run. The injection artifact is no longer visible, since it doesn't exist. Other than that change, it is virtually impossible to detect any differences in the TDS distribution between these figures.

This result confirms that the injection activity as modeled, which utilizes extremely conservative assumptions, has no discernable impact upon the position of brines near the Pecos River.

3. *That the implementation of the storage term in SUTRA calculations be explained.*

This has been done and the calculations are included as an attachment.

4. *That the implementation of the equivalent fresh-water head concept into SUTRA be explained.*

This has been done and the calculations are included as an attachment.

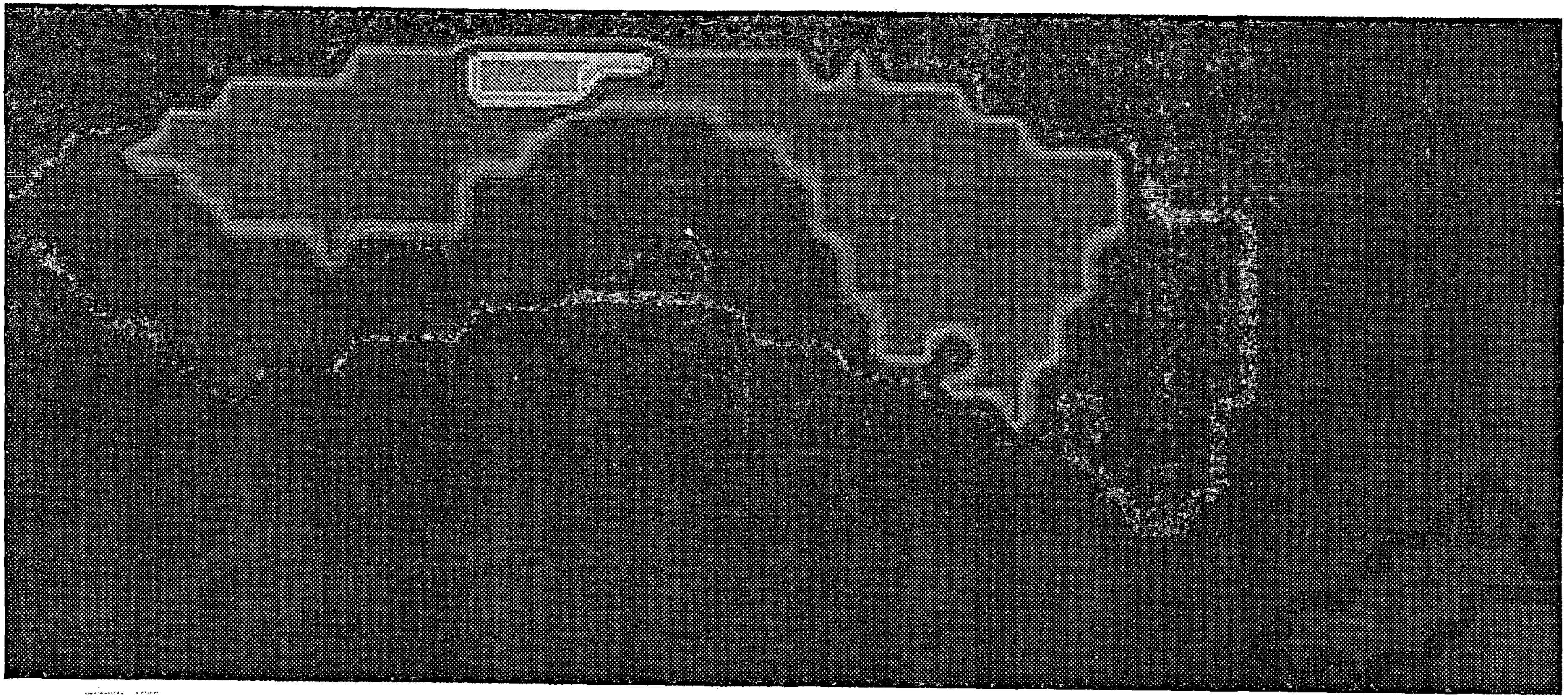


Figure 1. Brine Distribution after 50 Years of Injection Followed by 1,008 Years of No Injection (no regional gradient).

con0001_out vs. (row, col)

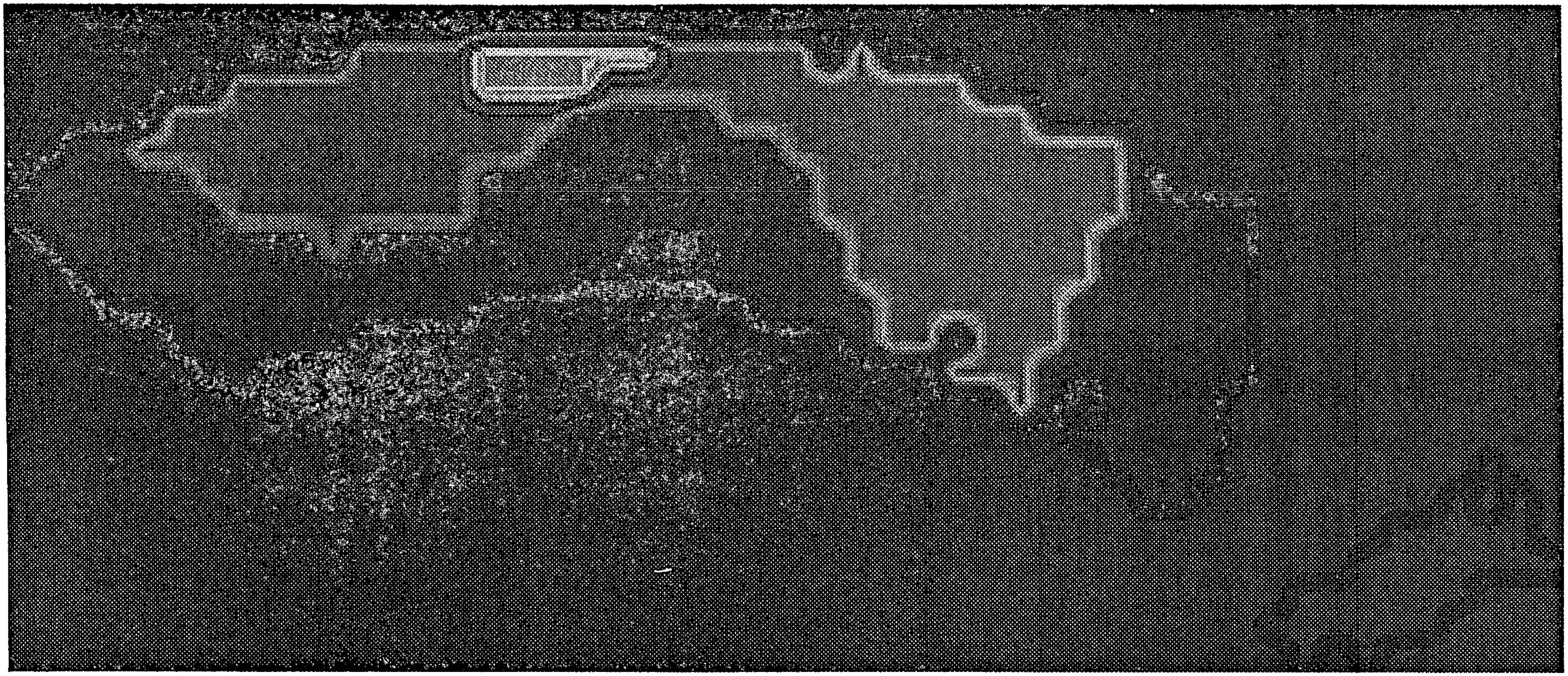


Figure 2. Brine Distribution after 1,058 Years. No Injection at any Time during Simulation (no regional gradient).

con0101_out vs. (row, col)

Storage Implementation of SUTRA

PURPOSE define the implementation of storage terms used in SUTRA. Compare this to conventional implementations, such as that of MODFLOW.

MODFLOW STORAGE:

MODFLOW utilizes the conventional storage factor definition. This definition is developed in ~~section~~ 2+5 of the manual.

Specific Storage is defined as S_s, the volume of water released from a unit aquifer volume per a unit change in head. It has dimensions of $\frac{L^3}{L^3 \cdot L} = \frac{1}{L}$

(L = Length)

Storage Coefficient is defined as S, the volume of water released from a unit aquifer surface per a unit drop in head. It has dimensions of $\frac{L^3}{L^2 \cdot L} = [D]$

([D] = dimensionless). In MODFLOW, practically speaking, it is equal to S_s multiplied by layer thickness. It is entered into the input files in that form.

MODFLOW has other variations of storage implementation. This above explanation only holds for the confined aquifer case. But that is also the case for the SUTRA runs in question, so no other storage implementations will be considered here.

SUTRA STORAGE:

In Freeze & Cherry (1979), Specific Storage is defined as:

$$S_s = \rho g (\alpha + n\beta) \quad (1)$$

M = mass
T = TIME

where
ρ = density of water $\left[\frac{M}{L^3}\right]$
g = acceleration due to gravity $\left[\frac{L}{T^2}\right]$
n = porosity of aquifer $[D]$
α = matrix compressibility $\left[\frac{L \cdot T^2}{M}\right]$
β = liquid compressibility "

Storage Implementation of SUTRA (cont.)

This term, though broken down into more factors, means exactly the same thing as the MODFLOW definition of specific storage.

On page 23 of the SUTRA manual, the specific pressure storativity, S_{op}, is defined:

> S_{op} = the volume of water released from saturated pore storage per unit aquifer volume due to a unit change in pressure.

$$S_{op} = \frac{\Delta VOL_{water}}{(VOL_{aquifer}) \cdot \Delta P} \quad (2)$$

Sop is related to Ss as follows:

$$S_s = \rho |g| S_{op} \qquad \left(\text{note that } \frac{P}{\rho g} = \text{pressure head}\right) \quad (3)$$

note: $|g|$ is magnitude of gravity vector component, dependant upon direction of elevation measurement.

equation 2.12 of SUTRA manual describes the release of mass of fluid from a aquifer due to a change in fluid pressure:

$$\frac{\partial (n\rho)}{\partial P} = \rho S_{op} \quad (4)$$

expanding equation above to account for matrix and fluid compressibilities:

$$\rho S_{op} = \rho \frac{\partial n}{\partial P} + n \frac{\partial \rho}{\partial P} \quad (5)$$

in SUTRA, β is expanded out as:

$$\beta = \frac{1}{\rho} \frac{\partial \rho}{\partial P} \quad (6)$$

and α is expanded out as:

$$\alpha = \frac{1}{VOL_{aquifer}} \cdot \frac{\partial (VOL)}{\partial \sigma'} \quad (7)$$

where $d\sigma' = -dp$ σ' is intergranular stress.

Storage Implementation in SUTRA (cont.)

also, this relation is derived in the manual:

$$\frac{\partial n}{\partial P} = (1-n)\alpha \qquad (8)$$

so that equation (5) can be re written as:

$$\rho S_{op} = \rho(1-n)\alpha + \rho n \beta \qquad (9)$$

factoring out density yields:

$$S_{op} = (1-n)\alpha + n\beta \qquad (10)$$

In the Capitan Models that are under review, the following values were "input":

$n = 0.18$

ρ = entered as 1.0 to effect the special output option of pressure output directly readable as heads.

assume for this exercise, that $\rho = 1000$ kg./m³

$\beta = 1.34E\text{-}6\ ft^{-1} = 4.47E\text{-}10\ \frac{m \cdot s^2}{kg}$

$$\left[4.47E\text{-}10 \frac{m \cdot s^2}{kg} \left(\frac{1\ ft.}{.3048\ m}\right)\left(\frac{0.4536\ kg}{lb(mass)}\right)\underset{(density)}{\left(\frac{62.4\ lb.(mass)}{ft^3}\right)}\underset{(gravity)}{\left(\frac{32.2\ ft}{s^2}\right)} = 1.34E\text{-}6\ ft^{-1} \right]$$

$\alpha = 2.99E\text{-}7\ ft^{-1} = 1E\text{-}10\ \frac{m \cdot s^2}{kg}$

$$\therefore S_{op} = (0.82)\left(1E\text{-}10 \frac{m \cdot s^2}{kg}\right) + (0.18)\left(4.47E\text{-}10 \frac{m \cdot s^2}{kg}\right)$$

$$= 1.6246E\text{-}10\ \frac{m \cdot s^2}{kg}$$

$$\therefore S_s = S_{op} \cdot \rho g = 1.59E\text{-}6\ m^{-1} = 4.86E\text{-}7\ ft^{-1}$$

Given aquifer thickness of 1000 ft,

$S = 4.86E\text{-}4 = \boxed{0.000486}$

Equivalent Fresh Water Head - conversion from pressure in SUTRA

SUTRA iterates on the components of hydraulic head, not on head itself. head can be divided up ~~into~~ an elevation and a pressure component:

$$H = \frac{P}{\rho g} + z$$

where:
- H = hydraulic head
- P = pressure
- ρ = fluid density
- g = gravity
- z = elevation

Since the Capitan model is perfectly flat, there is no elevation gradient. Therefore the z term drops out, and $H = \frac{P}{\rho g}$.

The fluid mass balance equation from SUTRA p.34 #2.24, accounting for strictly saturated, confined flow may be rewritten as:

$$\left(\rho S_{op}\right)\frac{\partial P}{\partial t} - \nabla\cdot\left(K'\cdot\left(\nabla P - \rho g\right)\right) = Q \qquad (1)$$

where:
- t = time
- S_{op} = specific pressure storativity
- Q = source/sink term, set = 0
- $K' = K^{*}\rho/\mu$ — K^{*} = permeability, μ = viscosity

let $z_0 = 0$ = mean sea level and assume $\rho = \rho(x,y)$, and flow in vertical direction is zero.

then $P(x,y) = P_0(x,y)$, and equation (1) reduces to:

$$\rho S_{op}\frac{\partial P_0}{\partial t} - \nabla\cdot\left(K'\cdot\nabla P_0\right) = 0 \qquad (2)$$

Equivalent Fresh Water Head - conversion from pressure in SUTRA (cont.)

let $P_0 = \rho^* g H^*$

where H^* is equivalent head of fluid with density ρ^* (above $z = z_0 = 0$)

then :
$$\frac{\partial P_0}{\partial t} = \rho^* g \frac{\partial H^*}{\partial t}$$

and
$$\nabla P_0 = \rho^* g \nabla H^*$$

$\therefore$

$$\rho S_{op} \left(\rho^* g \frac{\partial H^*}{\partial t} \right) - \nabla \cdot \left(K' \rho^* g \cdot \nabla H^* \right) = 0 \qquad (3)$$

note that $\rho^* g S_{op} = S_s$

$$\therefore \quad \rho S_s \frac{\partial H^*}{\partial t} - \nabla \cdot \left(K \rho \cdot \nabla H^* \right) = 0 \qquad (4)$$

divide both sides by ρ^*

$$\frac{\rho}{\rho^*} S_s \frac{\partial H^*}{\partial t} - \nabla \cdot \left(\frac{\rho}{\rho^*} K \cdot \nabla H^* \right) = 0 \qquad (5)$$

let $\rho' = \frac{\rho}{\rho^*} \cong 1$

then
$$\boxed{S_s \frac{\partial H^*}{\partial t} - \nabla \cdot \left(K \cdot \nabla H^* \right) = 0} \qquad (6)$$

which is the ground water flow equation in terms of equivalent fresh water head,