COMPARISON OF HOLDITCH & LEE
METHOD TO BOTTOM HOLE PRESSURE
BUILD UP ANALYSIS

VACA DRAW/PITCHFORK AREA
TGF DESIGNATION - MORROW SANDS

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ENRON OIL AND GAS COMPANY

VACA DRAW / PITCHFORK RANCH AREA
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Permeability Calculations
Comparison of Holditch & Lee Method
To Bottom Hole Pressure Build Up Analysis

lh128rcl

WELL NAME	BH BUILD UP	H&L (skin = 0)	H&L with skin
Andrikopoulos No. 1	0.1081 md; 1.738 skin	0.0945 md	0.1212 md; 1.7 skin
Bell Lake 2 State No. 1	0.0098 md; -3.05 skin	0.0295 md	0.0134 md; -3.10 skin
	0.0611 md; -2.40 skin	0.0983 md	0.0621 md; -2.40 skin
Longway Draw No. 1	0.0250 md; -0.60 skin	0.0429 md	0.0387 md; -0.60 skin

ITERATIVE METHOD OF ESTIMATING FORMATION PERMEABILITY AND STABILIZED FLOW RATE FROM TRANSIENT FLOW DATA

(S.A. Holditch and W.J. Lee)

Formation permeability and stabilized flow rate can be estimated from short-term, pre-stimulation flow tests in tight gas reservoirs. These formation and well properties are rarely measured directly; accordingly, there is a need to calculate them from the types of measurements that are made. The calculated properties can then be used to assist regulatory agencies in determining when specific formations qualify for special price incentives.

Permeability and stabilized rate estimation procedures proposed in this report are based on flow equations firmly grounded in recent research in gas flow in porous media. Application of the equations is straightforward, as examples in the report illustrate.

2.0 INTRODUCTION

The purpose of this report is to present and illustrate calculation techniques to (1) estimate formation permeability from transient flow data in low permeability gas wells and (2) to estimate stabilized flow rate in an unstimulated gas well from data obtained during the transient flow period.

The need to estimate formation permeability arises because coring and core analysis at insitu formation conditions are infrequent in most reservoirs. The need to calculate (rather than measure) stabilized flow rates in low permeability wells arises because it can require months or years for rates to stabilize, making measurements impractical. In fact, most of these wells must be stimulated before they can produce at economic rates; accordingly, pre-stimulation tests of significant duration are rare. Despite scarcity of data, however, knowledge of formation permeability and stabilized flow rate may be required for a reservoir to be classified as a "tight gas reservoir" and thus qualify for special regulatory treatment, such as price incentives.

The calculation techniques are stated and illustrated in the following sections of this report. The theoretical basis for the techniques is summarized in the Appendix.

3.0 FORMATION PERMEABILITY ESTIMATES

Formation permeability can be estimated from transient (unsteady-state) flow test data obtained from a low permeability gas well prior to stimulation. In the Appendix, we show that flow in a gas well at pressures greater than about 3000 psia can be modeled adequately by*

$$\frac{q_g}{\bar{p} - p_{wf}} = \frac{kh}{141.2\ B_{gi}\mu_i\ [\ln\ (\frac{r_d}{r_w}) - 0.75 + s']}$$

where

$$r_d = \left(\frac{kt}{376\phi\mu_i c_{ti}}\right)^{\frac{1}{2}},\quad t \leq 948\ \phi\mu_i c_{ti} r_e^{\,2}/k$$

and

$$r_d = r_e,\quad t > 948\ \phi\mu_i c_{ti} r_e^{\,2}/k$$

Strictly speaking, this equation is valid only for tests conducted at constant rate; however, it is an acceptable approximation when rate declines smoothly (rather than abruptly), as in production through a fixed choke[4]. For lower reservoir pressures, a similar equation written in terms of a difference in pressures squared is a better model; this equation is also discussed in the Appendix.

* A table of nonmenclature is provided at the end of this report.

Permeability can be estimated using an iterative technique based on a simple rearrangement of the basic equation:

$$k = \frac{141.2\ q_g\ B_{gi}\ \mu_i}{h\ (p_i - p_{wf})}\ [\ln (\frac{rd}{r_w}) - 0.75 + s']$$

Application of this equation for permeability estimation is illustrated in the following example.

Example: An unstimulated well in the Cotton Valley formation flowed at 100 MCF/D. The rate was measured at 6 hour flow time; flowing bottom hole pressure was estimated to be 3000 psia at this time.

Other formation and completion properties are assumed to be:

$\gamma_g = 0.65$	$h = 50$ ft
$T = 265°$ F	$Z_i = 0.983$
$p_i = 5200$ psia	$c_{gi} = 1 \times 10^{-4}$ psi^{-1}
$\phi_g = 0.045$	$\mu_{gi} = 0.0328$ cp
$r_w = 0.333$ ft	Spacing = 320 acres
	$B_{gi} = 0.691$ RB/Mscf

Estimate formation permeability from these data.

Solution: Our method will be (1) to assume a value of k and calculate a transient radius of drainage, r_d, from

$$r_d = (\frac{kt}{376\ \phi\mu_i c_{ti}})^{\frac{1}{2}} \qquad \text{or, } r_d \doteq \left(\frac{Kt}{376\ \phi_g\ u_i\ C_{gi}}\right)^{1/2}$$

(2) to calculate k from

$$K = \frac{141.2\ q_g B_{gi} \mu_i}{h(p_i - p_{wf})} \ [\ln(\frac{r_d}{r_w}) - 0.75 + s'] \ ;$$

(3) to repeat steps (1) and (2) until assumed and calculated values of k agree;

(4) to verify that flow is transient at 6 hour flow time by checking the inequality

$$t \leq 948\ \phi \mu_i c_{ti} r_e^2/k \qquad \text{or, } T = 948\ \phi g\ ui\ Cgi\ re^2/k$$

Additional assumptions will be required in this case before a permeability estimate is possible.

(1) s' = 0 for this well (negligible damage or stimulation).

(2) $\phi c_{ti} \cong \phi_g c_{gi}$ (almost always an adequate assumption in a well producing only gas).

(3) Effective drainage radius, r_e, found from $\pi r_e^2 = (43{,}560)(320)$ or $r_e = 2106$ ft.

Trial 1: Assume k = 0.01 md.

$$r_d = [\frac{(0.01)(6)}{(376)(0.045)(0.0328)(1\times10^{-4})}]^{\frac{1}{2}} = 32.9 \text{ ft}$$

$$k = \frac{(141.2)(100)(0.691)(0.0328)}{(50)(5200-3000)} [\ln (\frac{32.9}{0.333}) - 0.75 + 0] = 0.0112 \text{ md}$$

Calculated k is slightly greater than assumed k; at least one more trial will be required.

Trial 2: Assume k = 0.0112 md.

$$r_d = (32.9)(\frac{0.0112}{0.01})^{\frac{1}{2}} = 34.8 \text{ ft}$$

$$k = (0.00291) [\ln (\frac{34.8}{0.333}) - 0.75] = 0.0113 \text{ md}$$

Convergence is adequate; k = 0.0113 md. We can verify that flow is unsteady state by noting that

$$6 \text{ hr} < 948\ \phi\mu_i c_{ti} r_e^2/k = (948)(0.045)(0.0328)(1\times10^{-4})(2106)^2/0.0113$$
$$= 5.49 \times 10^4 \text{ hr}$$

Flow is transient, and will remain so for 5.49×10^4 hr (6.3 yr) -- which illustrates vividly why stabilized flow conditions are not likely to be obtained in the typical tight gas reservoir flow tests.

Note: The iterative procedure used in this example would best be applied in practice using a programmable calculator or computer.

4.0 STABILIZED FLOW RATE CALCULATION FROM TRANSIENT TEST DATA

Stabilized flow rate at a given pressure drawdown can be estimated from flow rate measured during transient conditions by taking the ratio of $q/(\bar{p} - p_{wf})$ from the transient and pseudo-steady-state equations. The result is

$$\frac{q_{gs}}{q_g(t)} = \frac{\ln\left(\frac{r_d}{r_w}\right) - 0.75 + s'}{\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s'}$$

where

$$r_d = (kt/376\ \phi\mu_i c_{ti})^{\frac{1}{2}} \equiv (kt/376\ \phi_g \mu_i c_{gi})^{\frac{1}{2}}$$

Application of this equation is illustrated by the following example.

Example: Estimate the stabilized rate at which the well described in the previous example could deliver gas with a drawdown in bottom hole pressure of 2200 psi.

Solution: The first step in the calculation procedure is to determine formation permeability, k, so that the transient radius of drainage, r_d, can be estimated. In this example, permeability has been determined by the iterative procedure to be 0.0113 md, and r_d is 35.0 feet.

The next step is to calculate the stabilized gas flow rate, q_{gs}, from the equation

NOMENCLATURE

Symbol	Definitions
B_{gi}	$5.04\ TZ_i/p_i$ = Gas formation volume factor evaluated at initial reservoir pressure, RB/Mcf.
$\overline{B}_g$	$5.04\ T\overline{Z}/\overline{p}$ = Gas formation volume factor evaluated at average reservoir pressure, RB/Mcf.
c_{gi}	Gas compressibility evaluated at initial reservoir pressure, psi^{-1}.
c_{ti}	Total system compressibility evaluated at initial reservoir pressure, psi^{-1}.
D	Turbulance or non-Darcy flow coefficient, D/Mcf
h	Net formation thickness, ft
$\overline{k}$	Formation permeability, md
p_D	Dimensionless pressure
p_i	Initial formation pressure, psi
p_{sc}	Standard-condition pressure (14.7 psi)
p_{wf}	Flowing bottom hole pressure, psi
q_g	Gas flow rate, Mcf/D
q_{gs}	Stabilized gas flow rate, Mcf/D
$q_g(t$	Transient gas flow rate, Mcf/D
r_d	$(kt/376\ \phi\mu_i c_{ti})^{\frac{1}{2}}$ = Transient radius of drainage, ft
r_e	Radius of drainage, ft
r_w	Wellbore radius, ft
s	Skin factor, dimensionless
s'	$s + D\|q_g\|$ = Apparent skin factor, dimensionless
t	Elapsed time, hr
T	Formation temperature, °R
T_{sc}	Standard-condition temperature (520° R)
Z_i	Gas law deviation factor elevated at initial reservoir pressure, dimensionless
γ_g	Gas gravity (air = 1.0)
μ_i	Gas viscosity evaluated at initial reservoir pressure, cp
ϕ	Formation porosity, fraction
ϕ_g	Gas porosity, fraction

$$q_{gs} = q_g\,(t)\,\frac{[\ln\,(rd/r_w) - 0.75 + s']}{[\ln\,(r_e/r_w) - 0.75 + s']}$$

In this case,

$$q_{gs} = \frac{100\,[\ln\,(35.0/0.333) - 0.75 + 0]}{[\ln\,(2106/0.333) - 0.75 + 0]} = 48.8 \text{ Mcf/D}$$

Thus, with the same drawdown (2200 psi) observed in the 6 hour test, the stabilized rate would be approximately 48.8 Mcf/D. Approximately 6.3 years would be required to achieve this rate, as calculated in the previous example.

Once stabilized rate is known at the pressure drawdown imposed in the transient flow test, stabilized rate at other drawdowns can be estimated from the relationship

$$q_{gs,2} = q_{gs,1}\,\frac{(\bar{p}-p_{wf})_2}{(\bar{p}-p_{wf})_1}$$

This relationship is approximately correct when the apparent skin factor, s', is negligible or when its dependence on rate is negligible.

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APPENDIX

The purpose of this Appendix is to present derivations of equations used to estimate permeability and stabilized flow rate in low permeability, high pressure gas reservoirs. These equations include a transient flow equation, an equation for transient radius of drainage, and a pseudo-steady-state flow equation.

Transient Flow Equation

Recent research[1,2] has shown that, for reservoir pressures above 3000 psia, gas flow in a reservoir is adequately modeled by the equation

$$p_{wf} = p_i - 50{,}300 \frac{Z_i \mu_i}{2p_i} \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} [p_D + s + D|q_g|] \quad (1)$$

For transient flow (i.e., flow in which the pressure drawdown has not yet been influenced by reservoir boundaries),

$$p_D = \frac{1}{2} \ln \left(\frac{kt}{1688\ \phi \mu_i c_{ti} r_w^2}\right) \quad (2)$$

Thus, for transient flow at high pressures,

$$p_{wf} = p_i - 50{,}300 \frac{Z_i \mu_i}{2p_i} \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[\frac{1}{2} \ln \left(\frac{kt}{1688\ \phi \mu_i c_{ti} r_w^2}\right) + s + D|q_g|\right] \quad (3)$$

This equation can be simplied with the following substitutions:

$$s' = s + D|q_g| \tag{4}$$

$$\frac{T_{zi}}{p_i} = B_{gi}/5.039 \tag{5}$$

The result, for T_{sc} = 520° R and p_{sc} = 14.7 psia

$$p_{wf} = p_i - \frac{141.2\, q_g B_{gi} \mu_i}{kh} \left[\tfrac{1}{2} \ln \left(\frac{kt}{1688\, \phi \mu_i c_{ti} r_w^2}\right) + s'\right] \tag{6}$$

It is convenient to define a transient radius of drainage, r_d, as[1]

$$r_d^2 = \frac{kt}{376\, \phi \mu_i c_{ti}} \tag{7}$$

In terms of this radius of drainage,

$$p_{wf} = p_i - \frac{141.2\, q_g B_{gi} \mu_i}{kh} \left[\ln (r_d/r_w) - 0.75 + s'\right] \tag{8}$$

For reservoir pressures below 2000 psia, gas flow in a reservoir is better modeled by the equation

$$p_{wf}^2 = p_i^2 - 50{,}300\ Z_i \mu_i \frac{p_{sc}}{T_{sc}} \frac{q_g T}{kh} \left[p_D + s'\right] \tag{9}$$

For transient flow, equation (2) still relates p_D to time; thus, for T_{sc} = 520° R and p_{sc} = 14.7 psia,

$$p_{wf}^2 = p_i^2 - 1422 \frac{q_g T \mu_i Z_i}{kh} \left[\tfrac{1}{2} \ln \left(\frac{kt}{1688\, \phi \mu_i c_{ti} r_w^2}\right) + s'\right] \tag{10}$$

which can also be written

$$p_{wf}^2 = p_i^2 - \frac{1422\ q_g T \mu_i Z_i}{kh} \left[\ln \left(\frac{r_d}{r_w}\right) - 0.75 + s'\right] \qquad (11)$$

For pressures between 2000 and 3000 psia, both equations (8) and (11) are somewhat inaccurate; traditionally, an equation in the "p^2" form similar to equation (11) has been used. At all pressure levels, flow is transient for $t \leq 948\ \phi \mu_i c_{ti} r_e^2/k$. Finally, we note that for brief-duration transient flow in a "new" (previously unproduced) portion of a reservoir, $\bar{p} \cong p_i$; thus, $\bar{p}$ can replace p_i in equations (8) and (11).

Pseudo-Steady-State Equation

For pseudo-steady-state flow (i.e., flow in which the pressure drawdown has reached the drainage radius of the well) in a cylindrical reservoir (well centered)[3],

$$p_D = \frac{0.0005274\ kt}{\phi \mu_i c_{ti} r_e^2} + \ln \left(\frac{r_e}{r_w}\right) - 0.75 \qquad (12)$$

For higher-pressure reservoirs, then, substituting into equation (11) and application of simplifications (4) and (5) gives

$$p_{wf} = p_i - \frac{0.07447\ q_g B_{gi} t}{\phi h r_e^2 c_{ti}} - \frac{141.2\ q_g B_{gi} \mu_i}{kh} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s'\right] \qquad (13)$$

Average drainage area pressure, $\bar{p}$, can be related to original reservoir pressure, p_i, with a material balance

$$p_i - \bar{p} = \frac{0.07447\ q_g \bar{B}_g t}{\phi h r_e^2 c_t} \cong \frac{0.07447\ q_g B_{gi} t}{\phi h r_e^2 c_{ti}} \tag{14}$$

Thus, the pseudo-steady-state equation can be written in the simplified form

$$p_{wf} = \bar{p} - \frac{141.2\ q_g B_{gi} \mu_i}{kh} [\ln (\frac{r_e}{r_w}) - 0.75 + s'] \tag{15}$$

For lower pressure reservoirs better described by the "p^2" equation, substitution of equation (12) into equation (9) gives

$$p_{wf}^2 = p_i^2 - \frac{0.750\ q_g T\ Z_i t}{\phi h r_e^2 c_{ti}} - \frac{1422\ q_g\ T\ \mu_i\ Z_i}{kh} [\ln (\frac{r_e}{r_w}) - 0.75 + s'] \tag{16}$$

Now the material balance can be written

$$p_i - \bar{p} = \frac{0.07447\ q_g \bar{B}_g t}{\phi h r_e^2 c_t} \cong \frac{0.375\ q_g\ T\ Z_i t}{\phi h r_e^2 c_{ti} (\bar{p} + p_i)/2} \tag{17}$$

or

$$p_i^2 - \bar{p}^2 \cong \frac{0.750\ q_g\ T\ Z_i\ t}{\phi h\ r_e^2\ c_{ti}} \tag{18}$$

Then, as an approximation,

$$p_{wf}^2 = \bar{p}^2 - \frac{1422\ q_g\ T\ \mu_i\ Z_i}{kh} [\ln (\frac{r_e}{r_w}) - 0.75 + s'] \tag{19}$$

Equations (15) and (19) are applicable for $t > 948\ \phi \mu_i c_{ti} r_e^2/k$.

Summary of Working Equations

The equations useful in applications for a gas well with $p > 3000$ psi are:

$$p_{wf} = \bar{p} - \frac{141.2\ q_g\ B_{gi}\ \mu_i}{kh} \left[\ln\left(\frac{r_d}{r_w}\right) - 0.75 + s'\right]$$

where

$$r_d = \left(\frac{kt}{376\ \phi\mu_i c_{ti}}\right)^{\frac{1}{2}}, \quad t \le 948\ \phi\mu_i\ c_{ti}\ r_e^2/k$$

and

$$r_d = r_e, \quad t > 948\ \phi\mu_i c_{ti} r_e^2/k$$

For a gas well with $p < 2000$ psi, the working equations are

$$p_{wf}^2 = \bar{p}^2 - \frac{1422\ q_g\ T\ \mu_i Z_i}{kh} \left[\ln\left(\frac{r_d}{r_w}\right) - 0.75 + s'\right]$$

where r_d has the same definition as above:

$$r_d = \left(\frac{kt}{376\ \phi\mu_i c_{ti}}\right)^{\frac{1}{2}}, \quad t \le 948\ \phi\mu_i c_{ti} r_e^2/k$$

and

$$r_d = r_e, \quad t > 948\phi\mu_i c_{ti} r_e^2/k$$

VACA DRAW/PITCHFORK RANCH AREA
TGS DESIGNATION

EXAMPLE CALCULATIONS
USING THE DIAMOND "6" FEDERAL NO. 1
PRE-STIMULATION FLOW DATA

Cgi = Gas compressibility evaluated at initial reservoir pressure, psi^{-1}

Pc = 675 psia T_c = 346° R (<u>from 4-pt Form C-122</u>)

Pr = 9,824 psia/675 psia = 14.6

Tr = $\frac{460 + 222}{346}$ = 1.97

Cgi = .034/675 = 5.04×10^{-5} spi^{-1} (From Fiq 6.10; Applied Petroleum Reservoir Engineering, Craft and Hawkins)

Bgi = Gas formation volume factor evaluated at initial reservior pressure, RB/Mcf

Bgi = 5.04 TZ_i/P_i

= (5.04)(682)(1.376)/9,824
= .482 RB/Mcf

kl813rcw

EXAMPLE: DIAMOND 6' FED No. 1

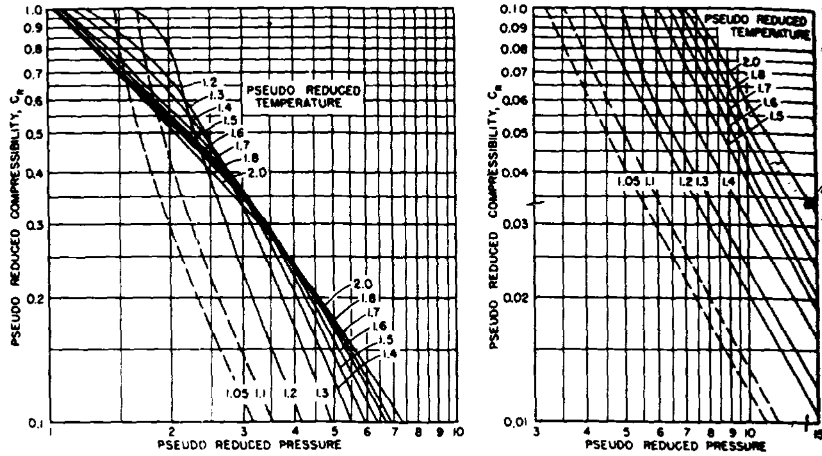

Fig. 6.10. The pseudoreduced compressibility of hydrocarbon gases as functions of their pseudoreduced temperatures and pressures. (*After* Trube,[5] *Trans.* AIME.)

pseudoreduced compressibility of a gas as a function of its pseudoreduced temperature and pressure. The actual compressibility is obtained by dividing the pseudoreduced compressibility by the pseudocritical pressure. Example 6.5 shows the use of Trube's curves.

Example 6.5. To find the compressibility of a 0.90 specific gravity gas condensate fluid at 150°F and 4500 psia using Fig. 6.10.

SOLUTION: From Fig. 1.2 find p_c = 650 psia and T_c = 427°R. Then

$$p_r = \frac{4500}{650} = 6.92 \text{ and } T_r = \frac{610}{427} = 1.43$$

From Fig. 6.10 find the pseudoreduced compressibility of 0.065 for p_r = 6.92 and T_r = 1.43. Then, since p_c = 650 psia,

$$c_g = \frac{0.065}{650} = 100 \times 10^{-6}\text{psi}^{-1}$$

(Compare with Example 6.4.)

In the study of transient flow in reservoirs the diffusivity constant $k/\mu c\phi$ enters the equations. So long as there is only one fluid present and rock compressibility is neglected, the compressibility is simply the compressibility of the fluid and the porosity is simply the *total* effective porosity. Where gas, oil, and water are present in the pore space, but only one of these three phases is mobile, the permeability is the *effective* permeability to that mobile phase and the viscosity is the viscosity of the mobile phase. In this case the product ($c\phi$) may be either (a) the product of the *average*

SEC. 3

compressibility
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mobile phase.
compressibility
compressibility

The *effective* co
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system above
ity, c_f, is gener
per psi. Wher
volume, it is
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Example 6.
($c\phi$).

Given:

ϕ = 0.1
S_g = 0.0
c_f = 7.5
c_g = 160

SOLUTION:

The product o

If oil is the n
effective comp

4. The C
systems are

-- Gas Viscosity --

Version 1.0

Well Name	DIAMOND 6 FED 1	21-Nov-91
Field Name	PITCHFORK	

Pressure	9,824 psia		
Reservoir Temp	222 'F	Z factor	1.376
Gas Gravity	0.580	Pressure/Z	7,141
Condensate (yes=1)	0	Gas Viscosity	0.03224
% N2	0.41 %		
% CO2	0.64 %		
% H2S	0.00 %		

-- BHP or Pwf Calculation --

Version 1.1
21-Nov-91

Well Name:	DIAMOND 6 FED 1			
Gas Gravity:	0.58		% N2	0.41
Condensate (yes=1):	0		% CO2	0.64 %
Reservoir Temp:	222	'F	% H2S	0.00 %
Surface Temp:	60	'F	Pc =	675.11 %
Depth of Zone:	15,250	feet	Tc =	350.67
Tubing Diameter:	2.350	inches		

FTP psia	Rate Mcfd	Pwf psia	Z	Pwf/Z psia
764	2,000	1,087	0.947	1,149
14	1,444	246	0.985	249
14	40,000	6,712	1.141	5,880