

V. S. Welch et al	36-16-30
(OPERATOR)	(S T R)
State B-2884	1
(LEASE)	(WELL NO.)

FORM

DATE APPROVED

C-101. . . .Depth 3000'	June 6, 1949
C-102. . . .Test casing.	
C-102. . . .Repair well	
C-102. . . .Deepen well	
C-102. . . .Shoot	
C-102. . . .Treat	
C-103. . . .Begin Drilling	
C-103. . . .Shoot	
C-103. . . .Treat	
C-103. . . .Test Casing	
C-103. . . .Deepen	
C-105. . . .	
C-110. . . .	

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For the first part, we need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$.

Let $f(x) = x^2$. Then $\int_0^1 x^2 dx = \frac{1}{3}$. We need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^2 = \frac{1}{3}$.

Consider the sum $\sum_{k=1}^n \left(\frac{k}{n}\right)^2 = \frac{1}{n^3} \sum_{k=1}^n k^2$. We know that $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$.

Therefore, $\frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^2 = \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6n^2}$.

As $n \rightarrow \infty$, $\frac{(n+1)(2n+1)}{6n^2} \rightarrow \frac{2}{6} = \frac{1}{3}$. Hence, $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^2 = \frac{1}{3}$.

For the second part, we need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$ for any continuous function f .

Let $f(x) = x^3$. Then $\int_0^1 x^3 dx = \frac{1}{4}$. We need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^3 = \frac{1}{4}$.

Consider the sum $\sum_{k=1}^n \left(\frac{k}{n}\right)^3 = \frac{1}{n^4} \sum_{k=1}^n k^3$. We know that $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$.

Therefore, $\frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^3 = \frac{1}{n^4} \cdot \left(\frac{n(n+1)}{2}\right)^2 = \frac{(n+1)^2}{4n^2}$.

As $n \rightarrow \infty$, $\frac{(n+1)^2}{4n^2} \rightarrow \frac{1}{4}$. Hence, $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^3 = \frac{1}{4}$.

For the third part, we need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$ for any continuous function f .

Let $f(x) = x^4$. Then $\int_0^1 x^4 dx = \frac{1}{5}$. We need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^4 = \frac{1}{5}$.

Consider the sum $\sum_{k=1}^n \left(\frac{k}{n}\right)^4 = \frac{1}{n^5} \sum_{k=1}^n k^4$. We know that $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$.

Therefore, $\frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^4 = \frac{1}{n^5} \cdot \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{(n+1)(2n+1)(3n^2+3n-1)}{30n^4}$.

As $n \rightarrow \infty$, $\frac{(n+1)(2n+1)(3n^2+3n-1)}{30n^4} \rightarrow \frac{6}{30} = \frac{1}{5}$. Hence, $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^4 = \frac{1}{5}$.

For the fourth part, we need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \int_0^1 f(x) dx$ for any continuous function f .

Let $f(x) = x^5$. Then $\int_0^1 x^5 dx = \frac{1}{6}$. We need to show that $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \left(\frac{k}{n}\right)^5 = \frac{1}{6}$.

Consider the sum $\sum_{k=1}^n \left(\frac{k}{n}\right)^5 = \frac{1}{n^6} \sum_{k=1}^n k^5$. We know that $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+5n+3)}{30}$.