

PREDICTION OF VERTICAL HYDRAULIC FRACTURE MIGRATION USING COMPRESSIONAL AND SHEAR WAVE SLOWNESS

R.M. NEWBERRY, R.F. NELSON, AND U. AHMED

ABSTRACT

Documented field results of vertical hydraulic fracturing suggest that quite often the created fracture migrates vertically away from the formation of interest (the hydrocarbon-bearing zone), thereby producing undesirable results. The single set of information needed to help answer questions concerning fracture migration consists primarily of in-situ stress, tensile strength, and elastic constants of the rock material in and around the formation of interest.

This paper describes the use of full waveform data from a sonic wireline tool to determine the relative stress distribution and the resultant induced hydraulic fracture height. Compressional and shear wave slowness, derived from the sonic waveforms, are used to calculate the dynamic elastic rock properties. A transversely isotropic model is used to compute the in-situ stress from the elastic properties. Advantages of the use of wireline measured data are discussed, as are the limitations of the technique. Final evaluation of the technique is shown through the comparison of predicted and poststimulation measured vertical fracture height. Two field cases are presented to illustrate the technique.

INTRODUCTION

With the recent advent of uncertain prices for oil and gas, the importance of efficiently developing hydrocarbon resources has increased significantly. Many costs are relatively fixed and cannot be greatly impacted by improved technology. However, new technology can make significant contributions in hydraulic fracturing operations. Often, a hydraulic fracturing treatment not only represents a large fraction of initial well costs, but also determines the economic viability of a particular well or field.

Too large a fracture treatment can be an unnecessary waste of completion funds, while too small a treatment may result in such inefficient drainage of the reservoir as to make a well unprofitable. Because of this economic double-edged sword, a hydraulic fracturing treatment must be designed which best exploits the reservoir.

Much research, both theoretical and applied, has been conducted in recent years toward greater understanding and control of fracturing treatments. Although several general three-dimensional computer-based models have been developed, their application has often been limited because of poor input data. Consequently, many rule-of-thumb schemes are often employed for local areas. When the proper input data are available, most of the more sophisticated models can predict the size and shape of a created fracture. Therefore, powerful tools exist in the industry for efficient fracture treatment design but have been underutilized for lack of sufficient field data. The consensus among investigators is that the information most needed for making realistic fracture geometry design decisions consists of the elastic parameters of the rock, the in-situ stress conditions, and the created fracture height.

In general, this information has been unavailable except on a relatively few research type operations. Therefore, data from a few wells are being extrapolated for use over large regions so that predictions from even the most sophisticated models often have a large degree of uncertainty. Usually, the greatest uncertainty about the created fracture geometry arises from the estimation of the vertical migration or height of the fracture with changes in treatment pressure and pumping conditions. A majority of hydraulic fracturing models require a realistic fracture height as an input.¹ Equations in fracture models also require elastic constants (Poisson's ratio and

moduli) in order to calculate fracture length and width. With information from wireline logs (compressional and shear wave travel time), it is now possible to quantify the changes in vertical fracture migration at the wellbore resulting from changes in treatment pressure, elastic values of formation rock, and boundary lithology. This paper shows the use of combining sonic waveform analysis with bulk volume analysis and treatment fluid density (1) to yield an accurate picture of the in-situ stress distribution in and around the hydrocarbon-bearing formation, and (2) to produce an accurate prediction of fracture migration behavior in relation to bottomhole treatment pressure.

COMPRESSIONAL AND SHEAR SLOWNESS

The heart of this procedure is based on the proper utilization of information contained in the wavetrain of an acoustic wave traveling through a section of the formation. In traveling through a section of rock, an acoustic pulse deforms the rock and, in turn, has its propagation characteristics altered by the rock. Measurements of the slowness of the compressional wave yield information about the reaction of the rock to a longitudinal stress. Likewise, the slowness of the shear wave is a measure of the reaction of the rock to a stress in the transverse direction. By combining this data with bulk density it is then a straightforward process to calculate Poisson's ratio and other elastic constants of the rock. The dynamic definition of Poisson's ratio is⁸

$$\gamma = [0.5 (v_c/v_s)^2 - 1] / [(v_c/v_s)^2 - 1] \dots (1)$$

and the dynamic definition of Young's modulus is

$$E = \rho_b v_s^2 [(3v_c^2 - 4v_s^2)/(v_c^2 - v_s^2)] \times 2.15 \times 10^8 \quad (2)$$

where

v_c = compressional slowness
 v_s = shear slowness
 ρ_b = bulk density of rock

Such measurements have been a standard laboratory procedure for several years. However, until recently it has been impossible outside the laboratory to measure accurately and consistently the slowness of the shear wave. Attempts to estimate the shear slowness based on lithology and compressional slowness have met with only limited success. The compressional wave travels through a formation at higher velocity than the shear wave. For proper detection of the shear arrival, the travel path of the wave must be sufficiently long to allow the earlier arriving compressional wave to dampen before the first shear arrival. Many of the

■ Mark of Schlumberger

newer sonic logging tools have receiver spacings such that the compressional and shear waves are sufficiently spread out in time to make routine detection of the shear wave possible (Fig. 1). Sophisticated computer analysis of these digitized wavetrains has provided the capability of rapidly extracting a shear slowness curve.¹⁰ After first motion is detected, automatic windowing of the waveform allows frequency domain operations to locate the shear wave arrival. This technique eliminates the need for an analyst to continuously decide visually what constitutes the first shear arrival from the various other modes of propagation. Fig. 2 presents an example of a Variable Density* log display of a typical section of openhole waveform data, showing the computer-located first motion and estimated compressional and shear arrivals before final processing. With measured values of bulk density and compressional and shear slowness available, it is then a straightforward process to calculate the in-situ stress, s_x , and the elastic constants which characterize the pay zone and boundary lithology. Appendix A details the derivation of the in-situ stress.

FRACTURE HEIGHT

Recent developments in the modeling of hydraulic fracture geometry have shown that the migration of hydraulic fractures is primarily dependent upon in-situ stress distribution and treatment pressure. Simonson et al.² has presented fracture equations that relate wellbore injection pressure to in-situ stress distribution, material properties, and fracture height migration. The successful use of the Simonson et al. concept to calculate variable height at the wellbore has been reported by Voegle et al.³ and Settari.⁴ Ahmed⁵ has shown how this concept can be applied to the hydraulic fracturing models of Geertsma and De Klerk⁶ and Perkins and Kern⁷ for design of optimized hydraulic fracture treatments. As depicted in Fig. 3, this fracture migration model has been modified to include fracturing fluid gravity effects and can be summarized in the following equations:

$$\Delta p = p_w - p_0 = c_1 [K_{ic} (\frac{1}{\sqrt{h_u}} - \frac{1}{\sqrt{h}})] \\ + c_2 (\sigma_b - \sigma_a) \cos^{-1} (\frac{h}{h_u}) \\ + c_3 \rho_f + (h_u - 0.5 h) \dots \dots \dots (3)$$

$$\Delta p = p_w - p_0 = c_1 [K_{ic} (\frac{1}{\sqrt{h_d}} - \frac{1}{\sqrt{h}})]$$

$$+ c_2 (\sigma_c - \sigma_a) \cos^{-1} \left(\frac{h}{h_d} \right)$$

$$- c_3 \rho_f (h_d - 0.5 h) \dots\dots\dots(4)$$

$$h_t = h_u + h_d - h \dots\dots\dots(5)$$

where

- A_p = net pressure above initial fracture extension pressure, $P_w - P_0$
- P_w = wellbore treatment pressure during fracturing
- P_0 = pressure required for fracture extension beyond initial breakdown
- h_u = fracture height migration into upper barrier
- h_d = fracture height migration into lower barrier
- h = pay zone gross height
- h_t = total fracture height
- k_{ic} = critical stress intensity factor
- σ_a = pay zone total matrix stress (fracture gradient pressure)
- σ_b = upper barrier total matrix stress (fracture gradient pressure)
- σ_c = lower barrier total matrix stress (fracture gradient pressure)
- ρ_f = fracturing fluid density
- c_1, c_2, c_3 = numerical constants

As shown in Appendix A, fracture gradient pressure at any particular depth can be expressed as a function of overburden pressure, pore pressure, Poisson's ratio, Biot's constant, and any unbalanced tectonic stress. Overburden pressure can be calculated by a simple integration of the bulk density; pore pressure can be measured by wireline tools or obtained by other formation-pressure measuring methods. In tectonically relaxed areas, unbalanced tectonic stress will normally produce only an offset to calculated stress values and, because of the technique of using differences in minimum horizontal stress, will not be an important consideration. Therefore, all the data needed to use the above model to relate net treatment pressure to fracture height migration is available. This procedure has been

implemented in a computer-generated product called the FracHite® log. An example of the log output is shown in Fig. 4. The following is a description of the data in each track.

Depth - Depth is labeled every 100 ft (30 m). Perforations are flagged.

Frac Height vs. Pressure - Two presentations are presented of how fracture height migrates at the wellbore with increase in treatment pressure above the fracture gradient pressure of the perforated zone. On the left is a step profile illustration of fracture height migration at discrete net pressures. The step profile is a realistic way of presenting the information. The step increase conforms with the ability of the fracture treatment service company to control pressure within 50 to 100 psi. The curve on the right presents increase in fracture height with continuous increase in net pressure. This curve is presented specifically to allow the user to see what is happening between steps and between zones and, also, to calculate fracture height for pressures within the specified steps as illustrated to the left. For this example, an increment of 350 psi was chosen. The first bar marks the height migration for a pressure increase of 350 psi; the second bar marks the migration for a 700-psi increase, etc.

Qualitative Fracture Geometry - Fracture geometry is related to treatment volume and conditions directly related to fracture costs. This two-dimensional format shows qualitatively how the fracture geometry changes with fracture height and treatment volume. It also allows the user to see where the treatment volume will go (boundary lithology or the hydrocarbon zone) and what relative volumes will be required to attain adequate fracture. This display is designed to allow the user to easily recognize the important considerations required in preparing an optimized hydraulic fracture treatment design. If problem areas are revealed, then a quantitative fracture design model should be used to "fine detail" the design. Such models are available from the fracture service companies.

Bulk Volume Analysis - The bulk volume presentation consists of the rock and its constituents (minerals, clays, porosity, and fluid saturations). There are two specific uses for this column: first, the selection of compatible fracturing fluid and second, identification of possible nearby aquifers.

Therefore, for a particular bottomhole treatment pressure, it is possible to determine the resulting upward and downward migration of the fracture. Conversely, for a particular height growth, it is possible to determine the maximum net treatment pressure (pressure above the average fracture gradient pressure of the perforated zone)

that can be applied and still remain within those bounds. This information is immediately useful in defining where barriers are located and quantifies the containment potential of each barrier. It is possible to quickly determine if sufficient containment exists for a particular fracturing treatment or to assess the risk of fracturing into nearby water zones. The application of this technique to wells containing multiple perforated intervals is described in another paper.⁹

In-situ stress in the perforated interval is treated as a single, lumped value. For the boundary lithologies, in-situ stress values are used every 2 ft (0.6 m) as they are encountered in moving up and down from the perforated interval. The contribution of hydrostatic pressure as the fracture migrates out of zone is included in the calculation by using an input value of borehole fluid density.

FIELD EXAMPLES

The example illustrated in Fig. 4 shows the results of the use of this technique on a section of the Cotton Valley formation in east Texas. In this zone, the calculations indicate that there is very good containment for net treatment pressures up to about 900 psi. By the time the pressure rises to 1050 psi, the height will have almost doubled, with much of the fracture treatment going into the side beds. For net pressures greater than about 1200 psi, the situation worsens and most additional fracture fluid goes into the adjacent shales. The perforated interval was 12021 to 12016 ft (3665 to 3662 m) and the maximum net treatment pressure was approximately 900 psi. For this net pressure, the predicted total height of the created fracture was 11 ft (3.5 m) from 12024 to 12013 ft (3665 to 3661.5 m). After-fracture monitor logs indicated the fracture extended from 12024 to 12012 ft (3665 to 3661.3 m) for a total height of 12 ft (3.7 m).

The second example (Fig. 5) demonstrates this technique on a section of the Blanco formation in Ojito County, New Mexico. Perforations were from 8068 to 8081 ft (2459 to 2463 m) and from 8110 to 8135 ft (2472 to 2480 m). For this well the maximum net bottomhole treatment pressure was approximately 1000 psi. The predicted total fracture height was 118 ft (36 m) from 8040 to 8158 ft (2451 to 2487 m). The after-fracture gamma ray log indicated the created fracture extended from 8039 to 8159 ft (2450 to 2487 m) for a total height of 120 ft (37 m).

COMPARISON OF METHODS

The primary benefit, from a scientific and practical viewpoint, of using wireline data for such an analysis is that continuous data are obtained. Measurements of cores or minifracture results normally yield only isolated, sporadic values. Also, minifracture data, normally confined

to only the pay intervals, give no information about the location and quality of the barriers. Economically, the cost of routinely coring long sections is usually prohibitive. Likewise, to gain minifracture information about the prospective barriers would require perforating, testing, and then squeezing off each tested interval. Such a procedure of actually fracturing the barriers would be an expensive and unwise method. This analysis is based on dynamically determined elastic constants, while hydraulic fracturing is considered a quasistatic process. The dynamic constants tend to be consistently higher than static measured values. Therefore, there should be a consistent offset to the calculations if static values are used instead of dynamic data. However, the fracture height prediction model uses only stress differences and not absolute values — with the result that any consistent shift has little effect on the calculations. Should core or minifracture data be available, it is a simple procedure to include this information, in effect, to calibrate the output to yield accurate absolute pressures.

CONCLUSIONS

The use of wireline acoustic data in conjunction with proper computer modeling provides an accurate and cost-effective means of obtaining important information needed for designing a hydraulic fracture treatment; i. e., fracture height, Poisson's ratio, and Young's modulus. Such data can be readily used by operators and service companies as input to their computerized fracture design programs.

NOMENCLATURE

v_c	= compressional slowness
v_s	= shear slowness
E	= Young's modulus
ρ_b	= bulk density of rock
ρ	= fracturing fluid density
Δp	= net pressure above initial fracture extension pressure
p_w	= wellbore treatment pressure during fracturing
p_o	= pressure required for fracture extension beyond initial breakdown
h_u	= fracture height migration into upper barrier
h_d	= fracture height migration into lower barrier
h	= pay zone gross height
h_t	= total fracture height
k_{ic}	= critical stress intensity factor
σ_a	= pay zone total matrix stress (fracture gradient pressure)
σ_b	= upper barrier total matrix stress (fracture gradient pressure)
σ_c	= lower barrier total matrix stress (fracture gradient pressure)
c_1, c_2, c_3	= numerical constants

ACKNOWLEDGEMENTS

We would like to acknowledge the contributions of Darrel Cannon, Vic Cook, and James Baker in the development and testing of this technique; and the assistance from the sales engineers, log analysts, and applications development engineers who provided field data and suggestions for this project.

REFERENCES

1. Veatch, R. W., Jr.: "Overview of Current Hydraulic Fracturing Design and Treatment Technology, Part 1 and Part 2," Soc. Pet. Eng. J. (April, May 1983).
2. Simonson, E. R., Abou-Sayed, A. S., and Jones, A. H.: "Containment of Massive Hydraulic Fractures," Soc. Pet. Eng. J. (Feb. 1978) 14, 1, 27-32.
3. Voegle, M. D., Abou-Sayed, A. S., and Jones, A. H.: "Optimization of Stimulation Design Through Use of In-Situ Stress Determination," paper SPE 10308 presented at the 1981 SPE Annual Technical Conference and Exhibition, San Antonio, October 5-7.
4. Settari, A.: "Quantitative Analysis of Factors Controlling Vertical Fracture Growth (Containment)," paper SPE 11629 presented at the 1983 SPE/DOE Low-Permeability Gas Reservoirs Symposium, Denver, March 13-16.
5. Ahmed U.: "A Practical Hydraulic Fracturing Model Simulating Necessary Fracture Geometry, Fluid Flow, Leak-Off and Proppant Transport," paper SPE 12880 presented at the 1984 SPE/DOE/GRI Symposium on Low-Permeability Gas Reservoirs, Pittsburgh, May 13-15.
6. Geertsma, J. and de Kleck, F.: "A Rapid Method of Predicting Width and Extent of Hydraulically Induced Fractures," J. Pet. Tech. (December 1969) 1571-1581; Trans., AIME, 255.
7. Perkins, T. K. and Kern, L. R.: "Widths of Hydraulic Fractures," J. Pet. Tech. (September 1961) 937-949; Trans., AIME, 22.
8. Howard, G. C. and Fast, C. R.: "Hydraulic Fracturing," Monograph Series, SPE, Dallas, 2.
9. Ahmed, U., Cannon, D. E., and Newberry, B. M.: "Hydraulic Fracture Treatment Design of Wells With Multiple Zones," paper SPE 13857 presented at the 1984 SPE/DOE Symposium on Low-Permeability Reservoirs, Denver, May.
10. Aron, J., Murray, J., and Seeman, B.: "Formation Compressional and Shear Interval-

Transit-Time Logging by Means of Long Spacings and Digital Techniques," paper SPE 7446 presented at the 1978 SPE Annual Technical Conference and Exhibition, Houston, October.

APPENDIX A

When pressure is increased in the borehole, rupture occurs in the plane that is perpendicular to the direction of least compressive stress. The pressure required to induce this fracture is called the initiation or breakdown pressure. Once a fracture has been started, the pressure necessary to hold the fracture open will be equal to the minimum total horizontal stress. This stress is often referred to as fracture gradient pressure and also as closure stress.

In tectonically relaxed areas, such as the Gulf Coast, the least principal stress is probably horizontal. In these areas, fracturing should occur along vertical planes. In tectonically compressed areas, the horizontal stresses may be greater than the overburden stress. In these areas where the least principal stress is vertical, horizontal fractures should be expected.

The following derivation illustrates these stress relationships in a tectonically relaxed area. In addition, an equation will be developed which can be used to estimate fracture gradients in areas where no prior data have been accumulated.

From Biot's theory of poroelastic deformation

$$s = \sigma + \alpha p \dots\dots\dots(A-1)$$

where

s = total stress gradient, psi/ft
 σ = matrix stress gradient, psi/ft
 p = pore pressure gradient, psi/ft
 α = Biot elastic constant

The Biot constant, α , can be expressed in terms of bulk compressibility and grain compressibility (the compressibilities of the rock with and without porosity).

In the vertical direction

$$s_z = \sigma_z + \alpha p = 1 \text{ psi/ft} \dots\dots\dots(A-2)$$

The horizontal matrix stress gradient (σ_x) can be related to the vertical matrix stress gradient (σ_z)

$$\sigma_x = \frac{\gamma}{1 - \gamma} (\sigma_z) \dots\dots\dots(A-3)$$

where

γ = Poisson's ratio

In the horizontal direction

$$s_x = \sigma_x + \alpha p \dots\dots\dots(A-4)$$

Substituting A-3 in A-4

$$s_x = \frac{\gamma}{1-\gamma} \sigma_z + \alpha p \dots\dots\dots(A-5)$$

Rearranging Eq. A-2 and substituting the vertical matrix stress gradient (σ_z) into Eq. A-5, the equation would become

$$s_x = \frac{\gamma}{1-\gamma} (s_z - \alpha p) + \alpha p \dots\dots\dots(A-6)$$

Eq. A-6 shows the minimum horizontal stress relationship for a homogeneous, isotropic, elastic material. In hard-rock areas, this model does not seem to adequately describe existing stress conditions. Another model of in-situ conditions is to assume the presence of microcracks in the rock. Such microcracks would tend to be aligned perpendicular to the least principal stress. Physically, the result of using such a model is that in the direction of least principal stress, it is appropriate to use

$$\alpha = 1 \dots\dots\dots(A-7)$$

and Eq. A-4 would become

$$s_x = \sigma_x + p \dots\dots\dots(A-8)$$

For such a transversely isotropic model, Eq. A-6 would become

$$s_x = \frac{\gamma}{1-\gamma} (s_z - \alpha p) + p \dots\dots\dots(A-9)$$

This model has provided results which are in close agreement with actual field data.

The calculation of fracture gradient in an area that cannot be classified as a tectonically relaxed basin is not straightforward. In basins containing numerous thrust faults, in areas near mountain ranges, or in basins which have been uplifted over geologic time, externally generated horizontal stresses can affect fracturing pressures. Eq. A-9 should be rewritten to include such cases

$$s_x = \frac{\gamma}{1-\gamma} (s_z - \alpha p) + p + p_{tec} \dots\dots(A-10)$$

Eq. A-10 is a more general form of the fracture gradient equation. The first two terms of the equation express the portion of the fracture gradient that is caused by the vertical stress gradient and the elastic properties of the rock. The third term, p_{tec} , includes the effects of externally generated horizontal forces. Normally, p_{tec} is determined empirically by using field data.

In tectonically relaxed and geologically simple areas, Eq. A-9 can be used to predict the fracture gradient. In the more complex geologic area, Eq. A-10 must be used and empirically calibrated by using field data from the specific geological region of interest.

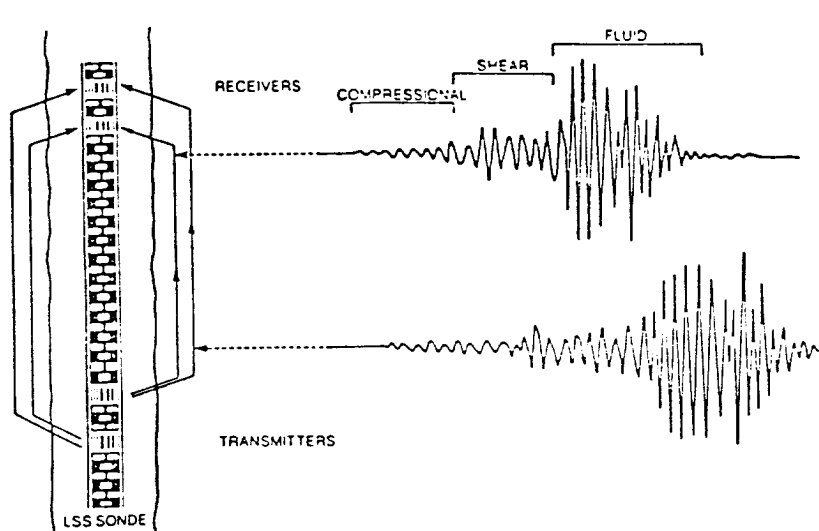


Figure 1—Typical Long-Spaced Sonic waveform pairs.

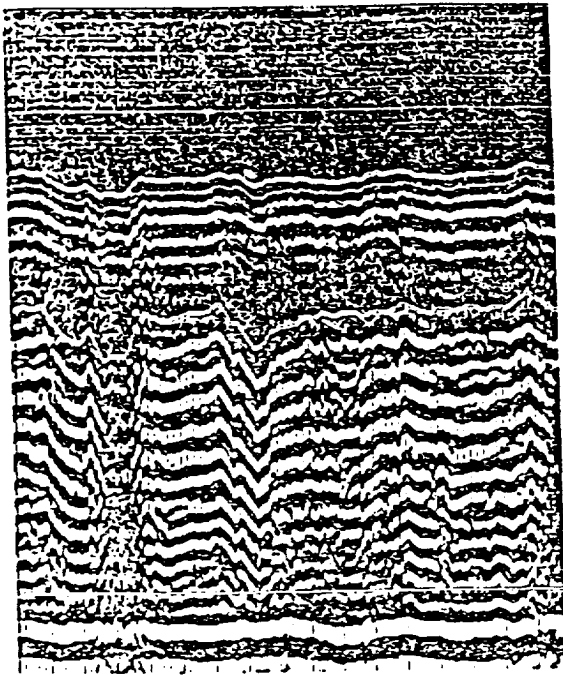


Figure 2—Variable density display of digitized waveform data.

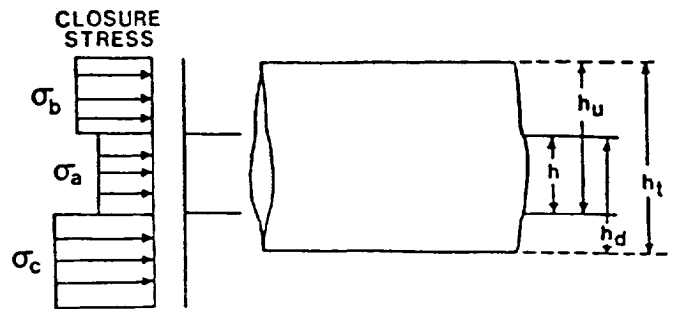


Figure 3—Simplified illustration of the fracturing model parameters (modified from Ahmed⁵).

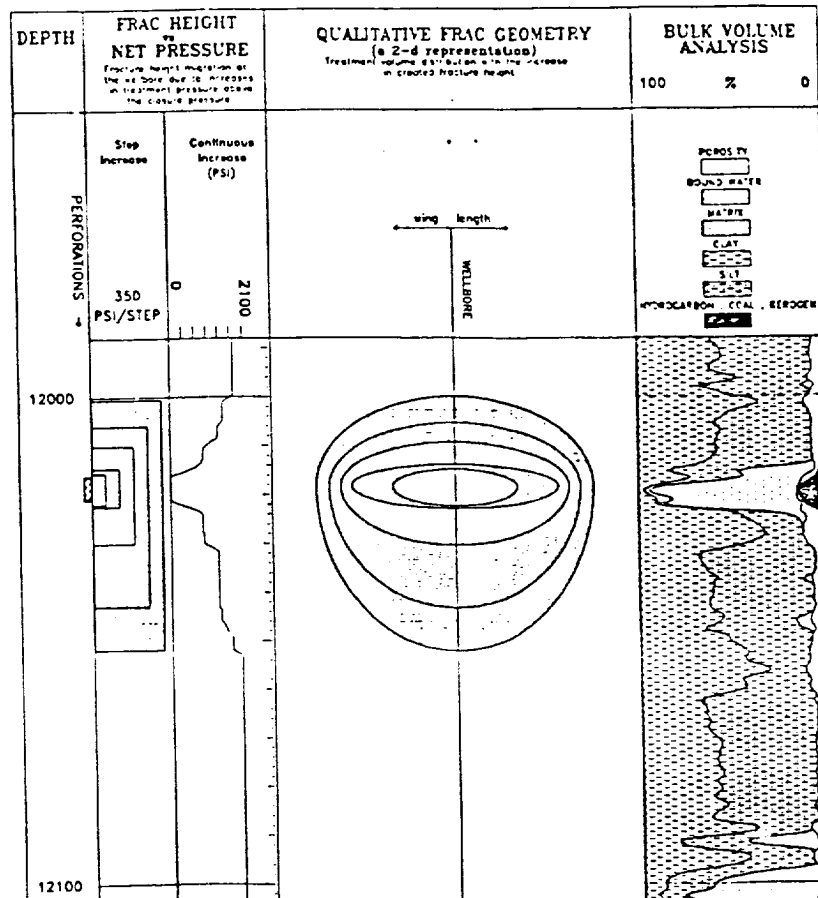


Figure 4—Typical FracHite log presentation.

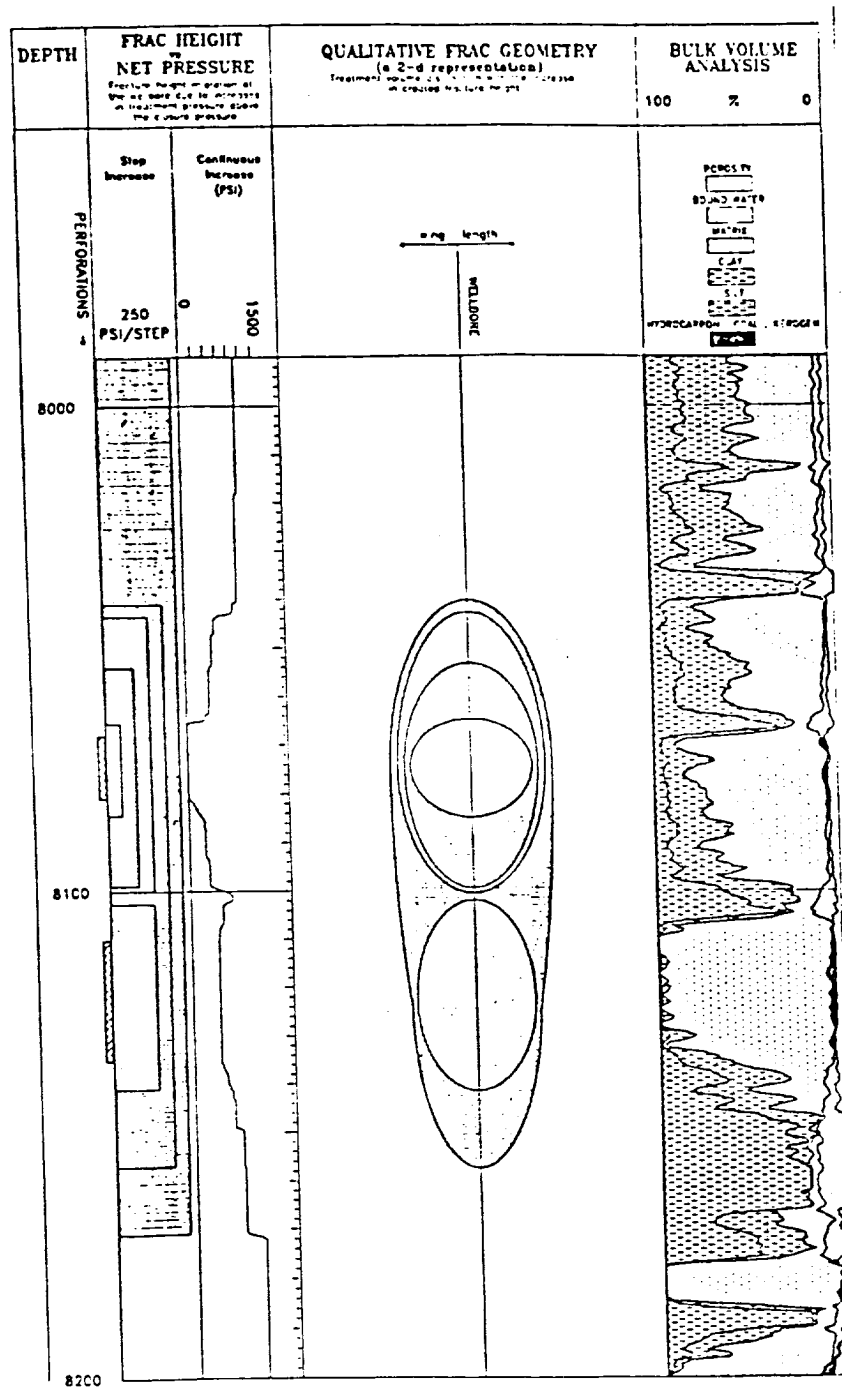


Figure 5—FracHite log on an example well from Ojito County, New Mexico.