How Permeability Depends on Stress and Pore Pressure in Coalbeds: A New Model

Ian Palmer, SPE, and **John Mansoori**, Amoco Production Co.

Summary

In naturally fractured formations such as coal, permeability is sensitive to changes in stress or pore pressure (i.e., changes in effective stress). This paper presents a new theoretical model for calculating pore volume (PV) compressibility and permeability in coals as a function of effective stress and matrix shrinkage, by means of a single equation. The equation is appropriate for uniaxial strain conditions, as expected in a reservoir. The model predicts how permeability changes as pressure is decreased (i.e., drawdown). PV compressibility is derived in this theory from fundamental reservoir parameters. It is not constant, as often assumed. PV compressibility is high in coals because porosity is so small. A rebound in permeability can occur at lower drawdown pressures for the highest modulus and matrix shrinkage values. We have also history matched rates from a boomer well in the fairway of the San Juan basin by use of various stress-dependent permeability functions. The best fit stress/permeability function is then compared with the new theory.

Introduction

During drawdown of a reservoir by primary production, effective stress increases and permeability decreases because of cleat compression. However, in coalbeds, drawdown leads to desorption of methane, and this is accompanied by matrix shrinkage, which opens the cleats and leads to permeability increase. The two effects of cleat compression and matrix shrinkage act in opposite directions on permeability.

The purpose of this report is to present a new theoretical formulation for stress-dependent permeability, which includes both stress effects and matrix shrinkage in a single equation. The equation is appropriate for uniaxial strain conditions, as expected in a reservoir. The new formulation also predicts PV compressibility, which is not constant, as is commonly assumed. This work is important in interpreting gas production behavior during drawdown. It may also have implications for enhanced recovery by gas injection.

Seidle *et al.*[1] measured PV compressibility from stress-dependent permeability experiments on cores in the laboratory. Here, we derive stress-dependent permeability from an equation that has the advantage that it applies to uniaxial strain, which is the usual condition in the reservoir, plus it combines cleat compression because of pore pressure falloff and matrix shrinkage because of gas desorption together in one equation. The matrix shrinkage term is a function of pore pressure and is incorporated in this way. Finally, we have attempted to history match rates from a boomer well in the fairway of the San Juan basin by incorporating various stress-dependent permeability functions.

Theory

The derivation starts from the following equation of linear elasticity for strain changes in porous rock. The incremental PV strain $d\varepsilon_p$ is a result of a simple volumetric balance between the bulk rock, the grains, and the pores,

$$d\varepsilon_p = \frac{d\varepsilon_r}{\phi} - \left(\frac{1-\phi}{\phi}\right)d\varepsilon_g. \quad \ldots\ldots (1)$$

The symbols are given in the Nomenclature. In this equation, changes in strain are assumed to be small (i.e., linear elasticity). Under uniaxial strain conditions, expected in reservoirs, the change in PV strain $d\varepsilon_p$ leads to a change in porosity as follows:[2,*]

$$-d\phi = \left[\frac{1}{M} - (1-\phi)f\gamma\right](dS - dp) + \left[\frac{K}{M} - (1-\phi)\right]\gamma dp - \left[\frac{K}{M} - (1-\phi)\right]\alpha dT, \quad \ldots\ldots (2)$$

where the compressibility of fluid in the pores is assumed to be very high (i.e., there is some gas in the pores), in which case, the fluid compressibility and fluid thermal expansivity terms drop out from the equation in Ref. 2. M (constrained axial modulus) and K (bulk modulus) are related to Young's modulus, E, and Poisson's ratio ν, through isotropic elasticity theory,[3]

$$\frac{M}{E} = \frac{1-\nu}{(1+\nu)(1-2\nu)} \quad \ldots\ldots (3)$$

and

$$\frac{K}{M} = \frac{1}{3}\left[\frac{1+\nu}{1-\nu}\right]. \quad \ldots\ldots (4)$$

For porosity, $\phi \ll 1$, as is the case in coalbeds, and for no change in overburden stress ($dS = 0$), we have

$$-d\phi = -\frac{1}{M}dp + \left[\frac{K}{M} + f - 1\right]\gamma dp - \left[\frac{K}{M} - 1\right]\alpha dT. \quad \ldots\ldots (5)$$

The term in dT = a temperature expansion/contraction term (if the temperature drops, the fabric shrinks, and the cleats increase in width). This is directly analogous to matrix shrinkage, where cleat width increases as gas desorbs during pressure drawdown. By direct analogy, for incremental rock volume strain (i.e., increase in strain per unit temperature or pressure change), we can write

$$\alpha dT \equiv \frac{d}{dP}\left(\frac{\varepsilon_\ell \beta p}{1+\beta p}\right)dp, \quad \ldots\ldots (6)$$

if we assume that the shape of the volumetric strain curve is a Langmuir curve of the form given by Eq. A-3 (see Fig. 12 of Ref. 5). This leads to

$$-d\phi = -\frac{dp}{M} + \left[\frac{K}{M} + f - 1\right]\gamma dp - \left[\frac{K}{M} - 1\right]\frac{d}{dP}\left(\frac{\varepsilon_\ell \beta p}{1+\beta p}\right)dp. \quad \ldots\ldots (7)$$

The moduli, M and K, are virtually independent of pressure, as can be seen from the laboratory results of Zheng *et al.*[5] (at least for the static case, which is appropriate for our work here). This leads to

$$d\phi = c_m dp + \varepsilon_\ell\left[\frac{K}{M} - 1\right]\frac{d}{dp}\left(\frac{\beta p}{1+\beta p}\right)dp, \quad \ldots\ldots (8)$$

where

$$c_m = \frac{1}{M} - \left[\frac{K}{M} + f - 1\right]\gamma. \quad \ldots\ldots (9)$$

This paper (SPE 52607) was revised for publication from paper SPE 36737, first presented at the 1996 SPE Annual Technical Conference & Exhibition, Denver, 6–9 October. Original manuscript received for review 25 October 1996. Revised manuscript received 17 August 1998. Paper peer approved 1 September 1998.

*Higgs, N.G., Amoco, Tulsa, Oklahoma, private communication, 1994.

Integrating and dividing by ϕ_0 leads to

$$\frac{\phi}{\phi_0}=1+\frac{c_m}{\phi_0}(p-p_0)+\frac{\varepsilon_\ell}{\phi_0}\left(\frac{K}{M}-1\right)\times\left(\frac{\beta p}{1+\beta p}-\frac{\beta p_0}{1+\beta p_0}\right). \quad (10)$$

Assuming that permeability varies with porosity as follows,[6]

$$\frac{k}{k_0}=\left(\frac{\phi}{\phi_0}\right)^3, \quad (11)$$

then the changes in permeability can be calculated as functions of elastic moduli, initial porosity, sorption isotherm parameters, and pressure drawdown. These equations are for one gas component only.

Eqs. 10 and 11 can be compared with those based on the standard approach with PV compressibility assumed constant,

$$\frac{\phi}{\phi_0}=\exp[c_p(p-p_0)] \quad (12)$$

and $$\frac{k}{k_0}=\exp[3c_p(p-p_0)]. \quad (13)$$

In the new theory, if matrix shrinkage is strong enough, the permeability will rebound at lower drawdown pressures. The pressure at which this occurs may be derived from Eqs. 10 and 11, and is given by

$$p_c=\left(\frac{0.48\varepsilon_\ell E}{\beta}\right)^{1/2}-\frac{1}{\beta}. \quad (14)$$

Note that this rebound pressure is independent of original reservoir pressure, p_0.

Sawyer *et al.*[7] have formulated an equation similar to Eq. 10. Although they use it in their reservoir simulator, they do not present any results on how permeability varies with pressure during drawdown.

PV Compressibility Under Uniaxial Strain Conditions

This is defined as

$$c_p=\frac{1}{V_p}\frac{dV_p}{dp}. \quad (15)$$

For coals in the San Juan basin, grain compressibility is virtually negligible compared with pore compressibility, and $\phi \leqq 0.5\%$. In this situation $c_p \approx c_\phi = (1/\phi)(d\phi/dp)$.

From Eq. 8,

$$c_p=\frac{c_m}{\phi}+\frac{\varepsilon_\ell}{\phi}\left[\frac{K}{M}-1\right]\frac{d}{dp}\left(\frac{\beta p}{1+\beta p}\right)$$

$$\approx\frac{1}{\phi M}+\frac{\varepsilon_\ell}{\phi}\left[\frac{K}{M}-1\right]\frac{\beta}{(1+\beta p)^2}, \quad (16)$$

with M and K/M functions of E and ν as given earlier.

In general, PV compressibility is a complex function of moduli, initial porosity, sorption parameters, and pressure drawdown. It is not constant, as often treated.[1,6] Even at early times, when the matrix shrinkage term is negligible in Eq. 16, PV compressibility is not constant because porosity ϕ can change significantly during drawdown because of natural fracture closure.

Application to the Field

We use $\varepsilon_\ell/\beta = 8$ to illustrate permeability rebound in the field. However, we note that there is an unresolved discrepancy between the matrix shrinkage parameter, $\varepsilon_\ell/\beta = 8$, and the only two direct laboratory measurements of ε_ℓ/β for methane by Seidle[8] (0.68) and by Harpalani[9] (1.35), as described in Appendix A. The matrix shrinkage constant, ε_ℓ/β, does not scale up because it is a grain property.

Table 1 contains the large-scale reservoir parameters expected for the San Juan basin. Scale-up of these parameters is discussed in Appendix A.

Fig. 1 shows the results of scale-up to the field in the San Juan basin. If we believe the porosity range in the field is 0.1 to 0.[illegible] then the permeability rebound depends crucially on E. For E [illegible] 4.45×10^5 psi, there is significant permeability rebound, and strong rebound for the lower porosity of $\phi_0 = 0.1\%$. This value of E corresponds to the mean of several PV compressibility measurements in the San Juan basin, as discussed in Appendix A.

In contrast, for the value of $E = 1.24 \times 10^5$ psi, permeability rebound is nonexistent, because stress effects dominate over matrix shrinkage. This value of E corresponds to the mean predicted by scaling core measurements of E up to the field. Which of the above two values of E is most appropriate to the field is not really known.

The results of Fig. 1 appear to be a good approximation if ϕ changes by less than a factor of 2, corresponding to permeability changes less than a factor of 10.

Fig. 2 shows how initial PV compressibility (i.e., neglecting matrix shrinkage) depends strongly on ϕ_0 and E in the field (from Eq. 16).

Discussion

Some caution needs to be applied in drawing conclusions about permeability rebound in the field.

- Permeability rebound depends most importantly on three parameters: ϕ_0, E, and ε_ℓ/β. It only appears in Fig. 1 for a combination of low ϕ_0 and high E.
- The value of $\varepsilon_\ell/\beta = 8$ illustrates permeability rebound seen in the field, although it has not been reconciled with independent measurements of matrix shrinkage for methane in the laboratory ($\varepsilon_\ell/\beta \approx 1$). However, under actual reservoir conditions, several factors might enhance the permeability rebound in the field.
- A 10% concentration of CO_2 is common in the fairway region of the San Juan basin. Because matrix shrinkage is stronger for CO_2 than for methane, a 10% concentration of CO_2 will increase the t[illegible] matrix shrinkage and enhance the permeability rebound show[illegible] Fig. 1.
- Coals might gradually become stiffer during drawdown (i.e., E might increase) because cleats are unable to close on asperities, coal fines, or mineralization. This would dilute the stress effects in Fig. 1 (i.e., the decrease in k/k_0 for initial drawdown might be suppressed). Clearly in a case like this, the permeability rebound would be enhanced.
- Coal failure can increase permeability rebound. When conditions are right, drawdown can induce failure because of changes in stress instigated by matrix shrinkage. Failure can lead to increased permeability through the phenomenon of dilatancy.

Our model may be compared with the earlier Seidle *et al.*[1] model. Seidle *et al.* have a constant 2 instead of $(K/M-1)$ in the shrinkage

TABLE 1—MATCH OF THEORY TO CORE MEASUREMENTS

Parameter	Large-Scale San Juan basin Reservoir
ϕ_0, %	0.1 through 0.5
ν	0.39
E, psi	$(1.24 \text{ through } 4.45) \times 10^5$
K/M	0.76
M/E	2.0
f	0.5
γ, psi^{-1}	0[illegible]
β, psi^{-1}	0.00[illegible]
ε_ℓ/β, psi	8

Application of Richardson Operating Co.
Record on Appeal, 949.

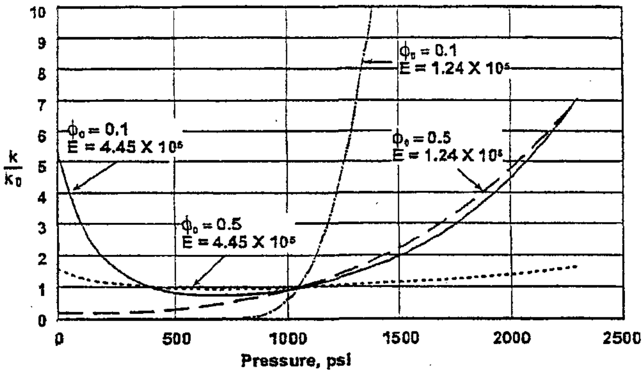

Fig. 1—Effect of pore pressure on coal permeability: $\varepsilon_l/\beta = 8$ and p_0 = 1,100 psi.

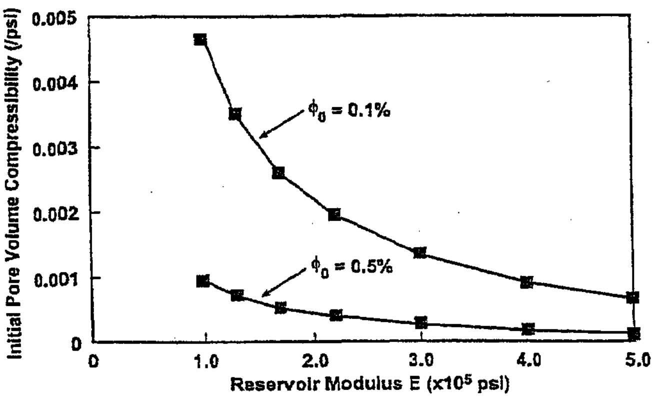

Fig. 2—Variation of initial PV compressibility (1,500 to 1,200 psi drawdown) with field values of ϕ_0 and E from Appendix A.

term (see Eq. 10 for example). This means that their shrinkage term is about 2.6 times greater than ours. Seidle *et al.* calculate ϕ/ϕ_0 separately for stress effects and shrinkage, then multiply them to get the combined effect. However, in our model they are added, not multiplied.

Finally, note that, in the Seidle theory, the PV compressibility is assumed to be constant with changes in pore pressure. But, as Eq. 16 and related discussion show, this is not true.

The new theory applies only to small changes in strain as demanded by linear elasticity (see Eq. 1). Because coal porosity is very low, porosity changes by up to a factor of 2 can be accommodated by the theory, corresponding to permeability changes less than a factor of 10.

History Matching of a Boomer Fairway Well

Figs. 3a through 3c show gas and water production and calculated bottomhole pressure (by use of casing pressure) for the San Juan fairway Well B1. The very strong gas production increase is representative of boomer fairway wells. This behavior is anomalous in that dewatering does not appear to explain the strong gas production. Furthermore, when casing pressure is reduced, as shown in Fig. 3c, there is a tremendous increase in gas production, more so than expected from Darcy's Law. One interpretation of this is that there is a rebound in permeability in the reservoir, and when the casing pressure is lowered, the permeability actually increases, reflecting this rebound curve.

An automatic history matching technique has been used to match the results in Fig. 3, by use of three empirical stress-permeability functions shown in **Fig. 4**. In Case 1, the permeability rebound is strong, and, at low drawdown pressures, the permeability actually exceeds the initial permeability in the reservoir. Case 3 represents no permeability rebound and uses a conventional exponential

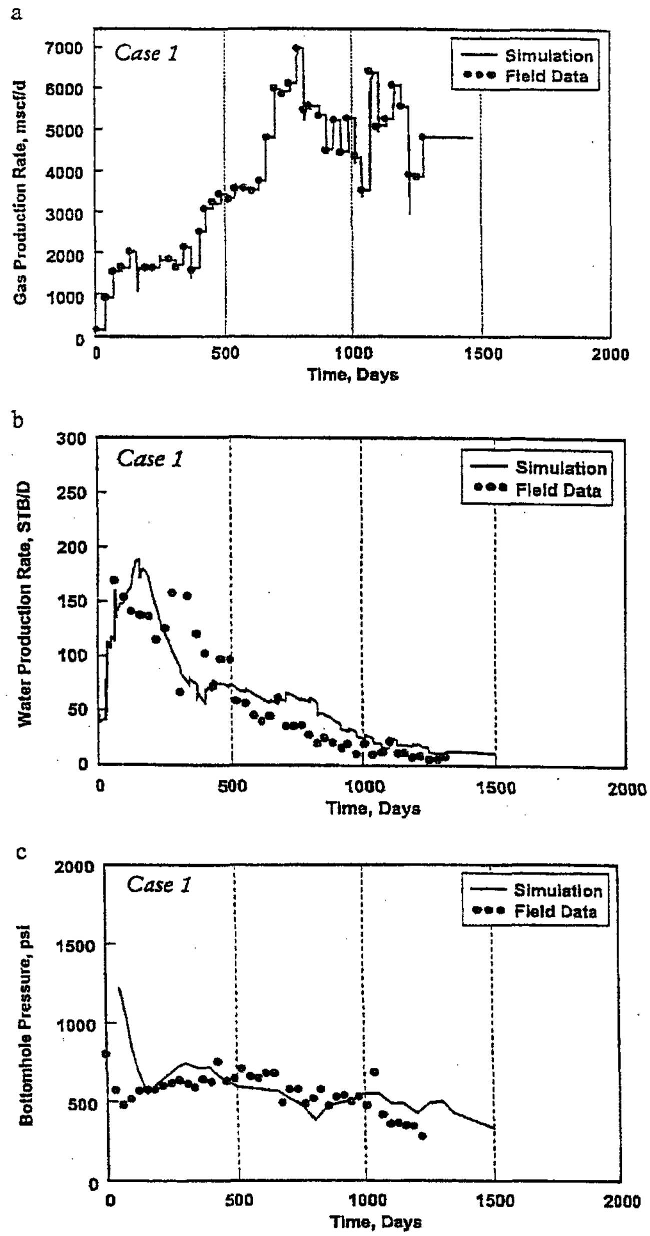

Fig. 3—Comparison of simulation and field data values for Case 1, Well B1. (a) gas production rate, (b) water production rate, (c) bottomhole pressure.

decrease. These curves are programmed into the simulator by use of a permeability-pressure lookup table.

Figs 3a through 3c and 5a through 5c show the comparison of these single-layer model results for two of the stress-dependent permeability functions and field data. The gas production rate was chosen as input to the simulator and is honored in all three cases. **Table 2** shows the corresponding values of history matching parameters. Clearly, the Case 3 match is ridiculous because of the high initial reservoir permeability (500 md). Both water injection rate and bottomhole pressures are matched quite well in Figs. 3a through 3c for Case 1 (strong permeability rebound) and very poorly in Case 3 (no permeability rebound). This indicates that, to achieve a reasonable match of the primary performance, it is necessary to include a mechanism by which absolute reservoir permeability is increased as the reservoir is depleted. This is needed to guarantee a much flatter bottomhole pressure profile consistent with field observations.

Application of Richardson Operating Co.
Record on Appeal, 950.

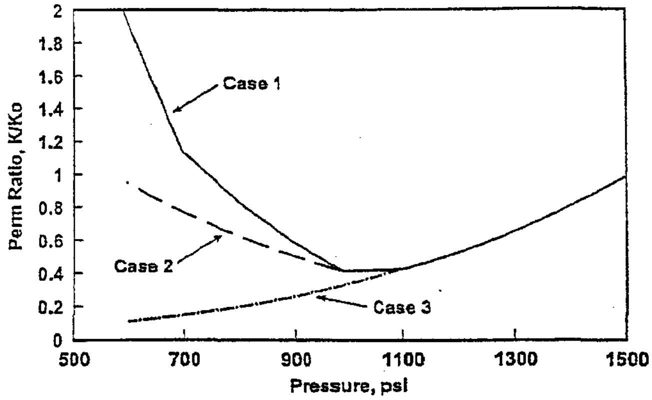

Fig. 4—Stress-dependent permeability models used to match Well B1 primary production.

The good match to the data by Case 1 in Fig. 4 uses a strong permeability rebound, with a minimum near 1,000 psi. We have not explored the sensitivity to shifts in this minimum. It also appears that a finer grid spacing would shift the minimum in the permeability curve more to the left. However, we suspect that any shift in minimum pressure would not be more than 300 psi. In conclusion, the pressure minimum for Case 1 in Fig. 4 is in quite good agreement with the two permeability rebound curves in Fig. 1.

However, there may be other interpretations for the well-production behavior in Fig. 3a. We have emphasized one interpretation (i.e., permeability rebound, and found that observations support the theory).

Conclusions

The theory we have developed shows how absolute permeability changes as reservoir pressure decreases during drawdown. However, the changes in permeability are strictly only applicable where the pressure is constant (static case), (i.e., they do not apply to the case when a gradient of pressure exists during flow toward the wellbore). In such a case, a fully coupled model (i.e., flow and stress are coupled) needs to be used to properly examine the changes in permeability during drawdown. Consequently, the history-matching example above from the San Juan basin, by use of a pseudocoupled model, is regarded as a first approximation only. A coupled model has been formulated by Durucan *et al.*,[10] but the model does not include matrix shrinkage (it does include stress-dependent permeability).

A model study on the effects of matrix shrinkage on permeability in coals has also been done by Levine.[11] The model appears to be less rigorous. Furthermore, for the field-scale parameters he has chosen, permeabilities always increase during drawdown, whereas, for our parameters in Fig. 1, we show two rebound curves and two nonrebound curves. The models agree in that the change in permeability is strongly dependent on the matrix shrinkage coefficient and the elastic modulus. However, our model indicates that porosity also is very important.

The theory we have developed above gives changes in natural fracture porosity because of pressure drawdown. This same thermoporoelastic formulation leads to stress changes. The total horizontal stresses are reduced because of lowering of pore pressure and matrix shrinkage (equivalent to a drop in temperature). Although transients occur, they lead to steady-state expressions for stress changes, which come out of the theory. In fact, the same terms that appear in the porosity change in Eq. 2 also appear in the equations for stress change. That is, the stress changes are implicit in the porosity changes (and permeability changes) in the theory. Horizontal stress changes because of matrix shrinkage do not have to be added separately because their effect is already included in the theory results of Eqs. 10 and 11.

Our conclusions may be summarized as follows.

1. We have developed a new theoretical formulation for stress-dependent permeability in the field. Stress effects and matrix shrinkage during drawdown appear naturally in a single equation. This equation is appropriate for uniaxial strain conditions expected in a reservoir.

2. Grain compression/expansion effects are included in the theory, but these are virtually negligible in the field.

3. The new formulation incorporates matrix shrinkage, not as an average, but as a function of reservoir pressure during drawdown.

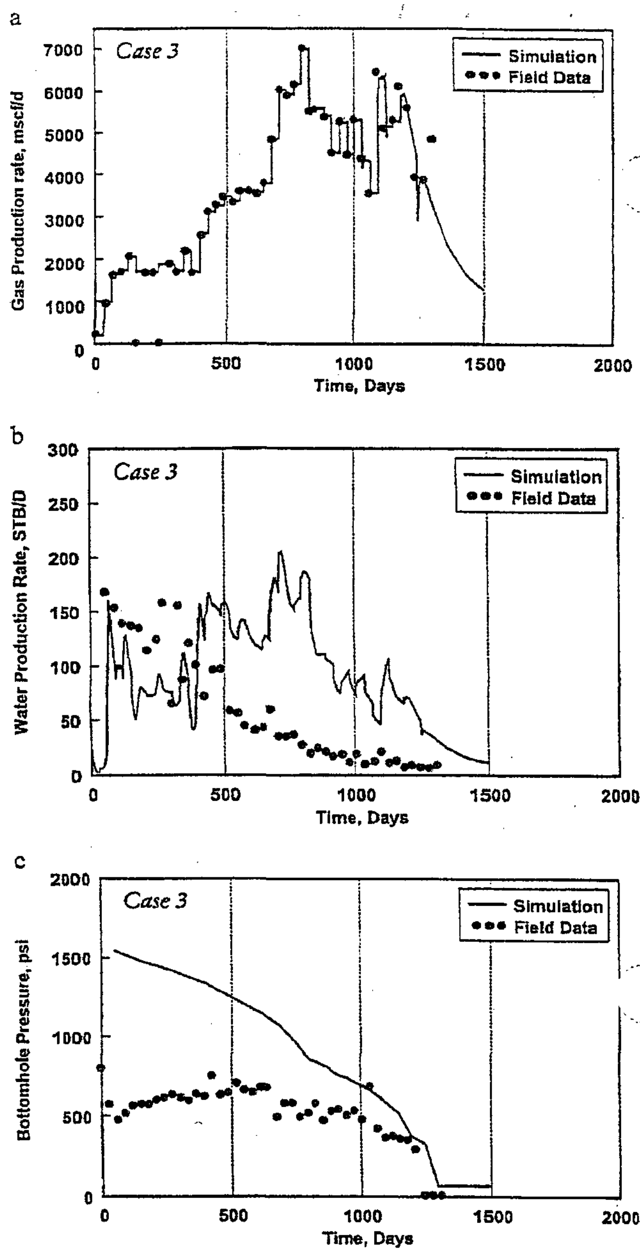

Fig. 5—Comparison of simulation and field data values for Case 3, Well B1. (a) gas production rate, (b) water production rate, (c) bottomhole pressure.

TABLE 2—HISTORY MATCH PARAMETERS FOR WELL B1

Case	k_0 (md)	ϕ_0 (%)	Sw_0
1	16	0.085	0.87
3	500	0.1	0.9

Application of Richardson Operating Co.
Record on Appeal, 951.

4. PV compressibility is predicted by the theory, and it is not constant. It depends on matrix shrinkage as well as effective stress parameters and changes with drawdown.

5. In the theory, porosity changes do not have to be small. The theory is probably okay for porosity changes less than a factor of 2, corresponding to permeability changes less than a factor of 10.

6. A prominent permeability rebound can occur in the field, as illustrated by the new model with $\varepsilon_\ell/\beta = 8$. Permeability rebound is more likely for a combination of low porosity and high Young's modulus in the coal.

7. Well B1 in the San Juan basin has been matched by a single-layer model with a permeability that strongly rebounds below 1,000 psi (cf. initial reservoir pressure = 1,500 psi). The minimum pressure, p_c = 1,000 psi, agrees quite well with permeability rebound curves derived from theory, with $\varepsilon_\ell/\beta = 8$. This is encouraging support for the theory.

8. However, the matrix shrinkage parameter, $\varepsilon_\ell/\beta = 8$, is greater than that found in the laboratory, $\varepsilon_\ell/\beta \approx 1$. This discrepancy is not resolved.

Nomenclature

c_p = pore volume compressibility, psi^{-1}
dp = change in pore pressure, psi
dS = change in overburden stress, psi
dT = change in temperature, °F
$d\varepsilon_g$ = incremental grain volume strain, dimensionless
$d\varepsilon_p$ = incremental pore volume strain, dimensionless
$d\varepsilon_r$ = incremental rock volume strain, dimensionless
E = Young's modulus, psi
f = a fraction $0 \rightarrow 1$
k = permeability, md
k_0 = virgin permeability, md
K = bulk modulus, psi
M = constrained axial modulus, psi
p = reservoir pressure, psi
p_0 = virgin reservoir pressure, psi
V_p = pore volume
V = core volume
α = grain thermal expansivity, °F^{-1}
γ = grain compressibility, psi^{-1}
ε_ℓ, β = parameters of Langmuir curve match to volumetric strain change because of matrix shrinkage (ε_ℓ = dimensionless, $\beta = psi^{-1}$)
ν = Poisson's ratio
ϕ = natural fracture porosity, fraction
ϕ_0 = porosity at virgin reservoir pressure, fraction

Acknowledgments

Amoco is gratefully acknowledged for their permission to publish this paper. Special thanks to Nigel Higgs for deriving the starting equations. Thanks are due to Bruce Gash, Don Duhrkopf, Zissis Moschovidis, Alan Pierce, K.V.K. Prasad, Raj Puri, John Seidle, Hans Vaziri, Richard Volz, and Dan Yee for their comments. Margie Meyer is thanked for superb document creation support.

References

1. Seidle, J.P., Jeansonne, M.W., Erickson, D.J.: "Application of Matchstick Geometry to Stress-Dependent Permeability in Coals," paper SPE 24361 presented at the 1992 SPE Rocky Mountain Regional Meeting, Casper, Wyoming, 18–21 May.
2. Bradley, J.S. and Powley, D.E.: "Pressure Compartments in Sedimentary Basins: A Review," *AAPG Memoir*, American Association of Petroleum, P. Ortoleva (ed.), Tulsa, Oklahoma, (1994), 3–26.
3. Jaeger, J.C. and Cook, N.G.: *Fundamentals of Rock Mechanics*, (third edition), Chapman and Hall, London, 1979.
4. Harpalani, S. and Schraufnagel, R.A.: "Influence of Matrix Shrinkage and Compressibility on Gas Production From Coalbed Methane Reservoirs," paper 20729 presented at the 1990 SPE Annual Technical Conference and Exhibition, New Orleans, 23–26 September.
5. Zheng, Z., Khodaverdian, M., and McLennan, J.D.: "Static and Dynamic Testing of Coal Specimens," paper 9120 presented at the 1991 Society of Core Analysts Fifth Annual Technical Conference, August.
6. McKee, C.R., Bumb, A.C., and Koenig, R.A.: "Stress-Dependent Permeability and Porosity of Coal," *Proc.*, Coalbed Methane Symposium, Tuscaloosa, Alabama (November 1987) 183.
7. Sawyer, W.K., Paul, G.W., and Schraufnagel, R.A.: "Development and Application of a 3D Coalbed Simulator," paper CIM/SPE 90-119, *Proc.* Petroleum Society CIM, Calgary (June 1990).
8. Seidle, J.P. and Huitt, L.G.: "Experimental Measurement of Coal Matrix Shrinkage Due to Gas Desorption and Implications for Cleat Permeability Increases," paper SPE 30010 presented at the 1995 SPE International Meeting on Petroleum Engineering, Beijing, China, 14–17 November.
9. Harpalani, S. and Chen, G.: "Estimation of Changes in Fracture Porosity of Coal with Gas Emission," *Fuel* (1995) **74**, 10, 1491.
10. Durucan, S. *et al.*: "Numerical Modeling and Verification of Stress and Pore Pressure Dependent Dynamic Permeability Behavior of Coal Seams Around Wellbores," Intergas Symposium, Tuscaloosa, Alabama (May 1995) 413.
11. Levine, J.R.: "Model Study of the Influence of Matrix Shrinkage on Absolute Permeability of Coal Bed Reservoirs," *Coalbed Methane and Coal Geology*, Gayer, R. and Harris, I. (eds.), Geological Society, Special Publication (1996) **109**, 197–212.
12. Jones, A.H., *et al.*: "A Review of the Physical and Mechanical Properties of Coal with Implications for Coalbed Methane Well Completion and Production," Geology and Coalbed Methane Resources of the Northern San Juan Basin, Colorado and New Mexico (ed. J.E. Fassett), Rocky Mtn. Assoc. Geologists, Denver (1988).

Appendix A—Ranges of Values for Reservoir Parameters

Table 1 of the text summarizes values of the large-scale parameters appropriate to Fruitland coalbeds in the San Juan basin. Note that large-scale mechanical parameters such as Young's modulus should be lower than those measured in cores in the laboratory. This is because of the presence of larger scale natural fractures. In contrast, the large-scale Poisson's ratio should be larger than core values. Here, we briefly discuss the derivation of each of the parameter ranges in Table 1.

Porosity. Porosity values lie in the range 0.1 to 0.5%. These values are from history matches of primary production in the San Juan basin.

Elastic Parameters. Young's modulus has been derived in two ways: from static core measurements and from measurements of PV compressibility in the field.

In the former case of the laboratory measurements, Jones *et al.*[12] provide a range of (3 to 7) $\times 10^5$ psi from core measurements. Taking the large-scale modulus to be about a factor of 4 less than the core measurements,*,** this gives

$$E_{ls} = (0.75 - 1.75) \times 10^5 \text{ psi.} \quad \text{(A-1)}$$

An average of several PV compressibility measurements in the San Juan basin gives $c_p = (23.3 - 96.9) \times 10^{-5}$ psi^{-1}. Now, PV compressibility can be approximated by just the first term in Eq. 16 in the text, for early times, and with $M = 2.0E$ from Table 1, and this leads to

$$c_p \cong \frac{1}{2E\phi_0}. \quad \text{(A-2)}$$

By use of the range of ϕ_0 defined previously, i.e., ϕ_0 = 0.1 to 0.5%, then gives, from Eq. A-2, $E = (1.72 - 7.15) \times 10^5$ psi, derived from PV compressibilities measured in the field. We used the mean of the core-derived results and the mean of the field-

*Schatz, J., consultant, private communication, 1994.
**Durucan, S., Imperial College, private communication, 1995.

Application of Richardson Operating Co.
Record on Appeal, 952.

derived results to define a range for $E_{ls} = (1.24 - 4.45) \times 10^5$ psi in Table 1.

Poisson's ratio measured on cores in the laboratory fall in a range of 0.27 to 0.4.[12] According to Schatz,* large-scale Poisson's ratio should be about a factor 1.15 greater than core measurements. This would give an average large-scale ν of 0.39.

The values of K/M and M/E are found from isotropic elasticity[3] by use of the equations in the text. By use of large-scale, $\nu = 0.39$ gives $K/M = 0.76$ and $M/E = 2.0$.

The grain (or fabric) compressibility is given as[4] $\gamma = 9 \times 10^{-7}$ psi^{-1}, and, in a separate measurement,[6] by $\gamma = 1.7 \times 10^{-6}$ psi^{-1}. Because this term does not affect the results very much, we can take a simple average of these two values, giving $\gamma = 1.3 \times 10^{-6}$ psi^{-1} (with a corresponding $f = 0.5$). Note that, for highly friable, disaggregated coal, the effective grain compressibility under shear is mitigated by slipping or jiggling of cleats, and this can be modeled by setting $\gamma = 0$ in Eq. 9. In contrast, less friable, more competent coals should have $f = 0.5$, allowing the full effect of the grain compressibility term (i.e., $\gamma \neq 0$).

Matrix Shrinkage Parameters. Two measurements with methane have been made on San Juan core (Seidle and Huitt[8] and Harpalani[9]). Both authors fit their volumetric expansion measurements by a Langmuir curve of the form

$$\frac{\Delta V}{V} = \varepsilon_\ell \frac{\beta p}{1 + \beta p}. \qquad \text{(A-3)}$$

*Schatz, J., consultant, private communication, 1994.

Both authors find $\beta = 0.002$ psi^{-1}. However, their values of ε_ℓ/β lie in the range 0.68 to 1.35, with Harpalani's measurement being the larger. Note that these values were derived by summing the strains measured in the three orthogonal directions because they are volumetric strains.

We have used a matrix shrinkage parameter $\varepsilon_\ell/\beta \approx 8$ to illustrate permeability rebound in the field. The discrepancy with laborat values, $\varepsilon_\ell/\beta \approx 1$, has not been resolved.

SI Metric Conversion Factor

psi × 6.894 757 E+00 = kPa

SPEREE

Ian Palmer works for Amoco Production Co. in Tulsa, Oklahoma, where his interests are in well stimulation and reservoir geomechanics. Previously, he was Manager of Geoscience Research at the Natl. Inst. for Petroleum and Energy Research. He holds a PhD degree in space physics from the U. of Adelaide, Australia. **John Mansoori** is a staff petroleum engineer at Amoco Production Co. in Denver, where he works on reservoir modeling and production optimization for coalbed methane gas in the San Juan basin. Previously, he worked on research and development in various enhanced recovery processes with Phillips Petroleum Co. in Bartlesville, Oklahoma, and Amoco Technology Center in Tulsa. Mansoori holds BS and MS degrees from the U. of Tehran, Iran, and a PhD degree from Illinois Inst. of Technology in Chicago, all in chemical engineering.

Application of Richardson Operating Co.
Record on Appeal, 953.