



THE UNIVERSITY OF CHICAGO

PHYSICS DEPARTMENT

PHYSICS 439

PHYSICS 439: QUANTUM MECHANICS
LECTURE 10: PERTURBATION THEORY
PART 1: DEGENERATE PERTURBATION THEORY

LECTURE 10: PERTURBATION THEORY
PART 2: DEGENERATE PERTURBATION THEORY

DATE: _____

1. Consider a system with a Hamiltonian $H = H_0 + H_1$, where H_0 is the unperturbed Hamiltonian and H_1 is a perturbation. Suppose the unperturbed system has a degenerate ground state with energy E_0 and n degenerate states. The perturbation H_1 lifts this degeneracy. We want to find the first-order corrections to the energy levels and the corresponding eigenstates.

2. In the case of a two-fold degenerate ground state, the first-order energy corrections are given by the eigenvalues of the matrix $W_{ij} = \langle \psi_i | H_1 | \psi_j \rangle$, where $|\psi_i\rangle$ are the unperturbed degenerate states. The eigenvalues are the roots of the secular equation $\det(W - E^{(1)}I) = 0$.

3. For a two-fold degenerate state, the secular equation is a quadratic equation in $E^{(1)}$. The solutions are $E^{(1)} = \frac{W_{11} + W_{22}}{2} \pm \sqrt{\left(\frac{W_{11} - W_{22}}{2}\right)^2 + |W_{12}|^2}$. The corresponding eigenstates are linear combinations of the unperturbed states.

4. The first-order corrections to the energy levels are $E_0^{(1)} = E_0 + E^{(1)}$ and the first-order corrections to the eigenstates are $|\psi_i^{(1)}\rangle = |\psi_i\rangle + \sum_{j \neq i} \frac{\langle \psi_j | H_1 | \psi_i \rangle}{E_0 - E_j} |\psi_j\rangle$.